Sets and Probability

Day 1: SP.1 Sets
Day 2: *Sets Quiz* (a little longer than the usual daily quizzes)
SP.2 Intro to Probability
Day 3: Quiz and SP.3 Independent and Dependent Events
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SP.2 Intro to Probability

Probability is the measure of how likely an event is to occur.

$$P(\text{event } A) = \frac{n(A)}{n(U)} = \frac{\text{number of outcomes favorable to } A}{\text{total number of possible outcomes}}$$

A is the event you want to occur.
U is the universal set of all possibilities.

1. A glass jar contains 6 red, 5 green, 8 blue and 3 yellow marbles. If a single marble is chosen at random from the jar, what is the probability of choosing a red marble? a green or a yellow marble?

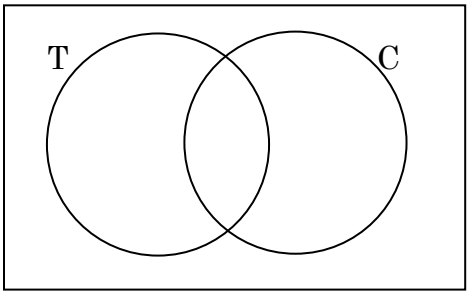
A "sample space" is a listing of all possible outcomes of an event. There are three common ways to show a sample space: lattice, tree, and Venn diagrams.

2. A red die and a white die are thrown. What is the probability that the sum of the numbers showing on the dice is 9 or 10? [lattice diagram]

Red / White	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

3. Suppose you toss a coin three times. What is the probability that exactly two of the tosses results in "heads"? [tree diagram]

4. In a class of 25 students, it is found that 16 of the students play tennis, 6 play both tennis and chess, and 3 do not participate in any activities at all.
   a) Find the probability that a student plays chess. [Venn diagram]



   b) Find the probability that a student does not play chess.

The probability of an event NOT occurring is called the "complement" of the event. It is written P(A'). We might say that P(not A) = 1 – P(A). Why?

5. A collection of 38 computer disks contain four that are defective. One is chosen at random. What is the probability it's defective? it's not defective?

<u>Classwork Exercises</u>

a) A card is chosen from a well-shuffled deck. What is the probability that it is a black ace? not a black ace? a diamond face card? a jack or a king?
b) If two dice are rolled, what is the probability that both show the same number?
c) If the probability of rain tomorrow is $\frac{2}{5}$, what is the probability of no rain?
d) Mr. & Mrs. Smith each bought 10 raffle tickets, and each of their three children bought 4 tickets. If 4280 tickets were sold in all, what is the probability that the grand prize winner is Mr. or Mrs. Smith? one of the 5 Smiths? none of the Smiths?
e) From a group consisting of Amal, Bara, Cesar, and Denay, two people are to be randomly selected to serve on a committee. List the sample space for this experiment. Find the probability that Bara and Cesar are selected. Find the probability that Cesar is <u>not</u> selected.
f) Of the 1260 households in a small town, 632 have dogs, 568 have cats, and 114 have both types of pets. Construct a Venn Diagram. If a household is chosen at random, what is the probability that the household has neither a cat nor a dog?
g) A bag has 20 discs numbered from 1 to 20. A disc is drawn at random . What is the probability that the disc has a number that is divisible by 2 and 5?
h) A number between 100 and 999, inclusive, is chosen at random. What is the probability that it contains at least one zero? no zeros?
i) A bag contains 7 pennies, 4 nickels, and 5 dimes. What is the probability of choosing a nickel? of not choosing a penny?
j) A family has 3 children. List the sample space, assuming birth order matters. What the probability that there are 2 boys and 1 girl? that there are 3 boys?

SP.3 Independent and Dependent Events

Today we will look at the probability of two events both happening. The notation for this is P(A and B) or $P(A \cap B)$. Tree diagrams are often the easiest way to organize these kinds of events.

Two events are <u>independent</u> if the outcome of the first event does not influence the outcome of the second event. The following formula applies:

$$P(A \cap B) = P(A) \times P(B)$$

1. Find the probability of getting a sum of 5 on the first toss of two dice and a sum of 3 on the second toss.

2. Andrew is 55, and the probability that he will be alive in 10 years is 0.72. Ellen is 35, and the probability that she will be alive in 10 years is 0.92. What is the probability that a) they will both be alive in 10 years; b) neither will be alive in 10 years; c) one of them will be alive in 10 years.

3. It is given that $P(A) = 0.6$, $P(B) = 0.7$ and $P(A \cap B) = 0.4$. Draw a Venn diagram to represent this information. Are events A and B independent?
   Why or why not?

4. It is given that $P(A) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. It is known that events A and B are independent. Find P(B).

Two events are <u>dependent</u> if the outcome of the first event in some way influences the outcome of the second event. The following formula applies:

$$P(A \cap B) = P(A) \times P(B \text{ given } A) = P(A) \times P(B \mid A)$$

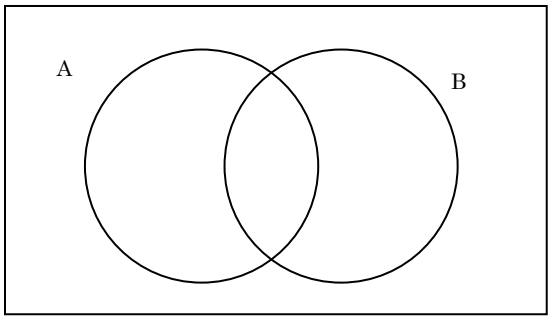
5. There are 2 cans of root beer and 4 cans of Dr. Pepper on the counter. Nada drinks two of them at random. What is the probability that she drank one can of each?

6. A quality-control procedure for testing Ready-Flash disposable cameras consists of choosing two cameras at random from each lot of 100 without replacement. If both cameras are defective, the entire lot is rejected. Typically, 10 cameras of the 100 are defective. Find the probability that the lot of cameras will NOT be rejected.

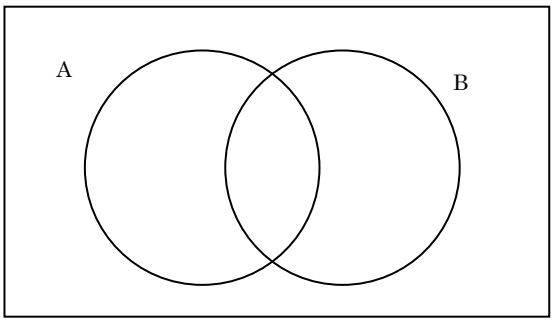
Probability + Set Theory

Problems involving probability are often solved by using Venn Diagrams:

1. Given P(A) = 0.55, $P(A \cup B)$ = 0.7, and $P(A \cap B)$ = 0.2, find $P(B')$

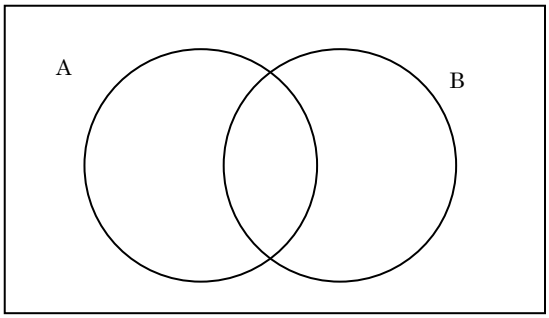


2. Given P(A) = $\frac{3}{5}$, P(B) = $\frac{2}{3}$, and $P(A \cap B)$ = $\frac{1}{2}$, find $P(A \cup B)$



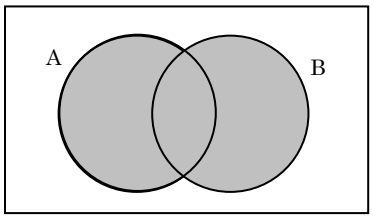
3. Given that P(A) = 0.6, P(B) = 0.7, and that A and B are independent events, find $P(A \cup B)$, $P(A')$, $P(A' \cap B)$.

Hmmm… What does "independent" mean?
What formula did we learn?



SP.4 Combined and Mutually Exclusive Events: $P(A \cup B)$

Two either/or events are <u>combined</u> if there exists the possibility that both events might occur at the same time. We have to consider the possibility of A occurring by itself, B occurring by itself, and both A and B occurring together. Because there is an area of overlap, we have to avoid double counting by subtracting the area of overlap.



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

1. When choosing a card, what is the possibility of choosing a king or a diamond?

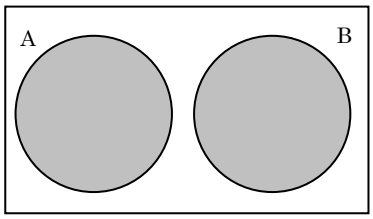
2. Professor Jackson is in charge of a program to prepare people for a high school equivalency exam. Records show that 80% of the students need work in math, 70% need work in English, and 55% need work in both areas. Compute the probability that a student selected at random needs help in math or English.

3. In a bag are 100 discs numbered 1 to 100. A disc is selected at random from the bag. Find the probability that the number on the selected disc is even or a multiple of 5.

4. A garage knows that when a person calls to report that their car won't start, the probability that the engine is flooded is 0.5 and the probability that the battery is dead is 0.4, and the probability of both is 0.1. What is the probability that the next person who calls will have either a flooded engine or a dead battery? Are these events dependent or independent? Why?

5. In a class, half the pupils study Mathematics, a third study English, and a quarter study both Mathematics and English. Find the probability that a student selected at random studies either Mathematics or English.

A special case of combined events occurs if either event may occur, but both cannot happen at the same time. We call these <u>mutually exclusive</u> events. In this case, there is no overlap, so we don't have to worry about double counting. It's the same equation as the one we saw on the previous page, but with $P(A \cap B) = 0$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = P(A) + P(B)$$

6. When tossing a die, what is the probability of tossing a 3 or a 4?

7. When choosing a card, what is the probability of choosing a jack or a king?

8. The Cost Less Clothing Store carries "seconds" in slacks that don't quite fit. If you buy a pair of slacks in your regular waist size without trying them on, the probability that the waist will be too tight is 0.30 and the probability that it will be too loose is 0.10. What is the probability that the waist won't fit?

9. Given that events A and B are mutually exclusive with P(A) = $\frac{3}{10}$ and P(B) = $\frac{2}{5}$, find the value of $P(A \cup B)$.

These two situations are easy to get confused. Be sure you know there are two possible formulas for $P(A \cup B)$:

Combined events:	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events:	$P(A \cup B) = P(A) + P(B)$

SP.5 Conditional Probability

Conditional probability measures the probability of an event A occurring <u>given that</u> B has already occurred. There is a formula for this in your packets. Sometimes it can be confusing for solving real-life problems…

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

In conditional probability, what you are doing is reducing the sample space to B and then finding how much A there is in B. Venn, tree, and lattice diagrams/charts can help **A LOT**…

1. Two dice numbered one to six are rolled. Find the probability of obtaining a sum of five given that the sum is seven or less. P(sum of 5|sum of 7 or less)

2. In a class of 25 students it is found that 5 of the students play both tennis and chess, 10 play tennis only, and 3 do not participate in any activities. Find the probability that a student selected at random from this group plays tennis, given the student plays chess.

3. Bag A contains 5 blue and 4 green marbles. Bag B contains 3 yellow, 4 blue, and 2 green marbles. Given you have a green marble, what is the probability it came from Bag A?

4. At the basketball game, Amanda got into a two-shot foul situation. She figured her chance of making the first shot was 0.7. If she made the first shot, her chance of making the second shot was 0.6. However, if she missed the first shot, her probability of making the second shot was only 0.4. Given Amanda missed the second shot, find the probability that she made the first shot.

5. The events A and B are independent. $P(A \cap B) = 0.6$ and $P(B) = 0.8$.
   a) P(A) b) P(A|B) c) P(A|B')