2-5/2-6 Postulates and Properties/Algebraic Proofs

A **postulate** or **axiom** is a statement that is accepted as true.

2.1 Through any two points there is exactly one line

2.2 Through any three points not on the same line, there is exactly one plane

2.3 A line contains at least 2 points

2.4 A plane contains at least three points not on the same line

2.5 If two points lie in a plane, then the entire line containing those points lies in that plane.

2.6 If two lines intersect, then their intersection is exactly one point

2.7 If two planes intersect, then their intersection is a line.

Example 1: Determine whether each statement is *always, sometimes, or never true.* Explain.

a. If points *A*, *B,* and *C* lie in plane *M*, then they are collinear.

b. There is exactly one plane that contains noncollinear points *P, Q,* and *R.*

c. There are at least two lines through the points *M* and *N*.

A **proof** is a logical argument in which each statement you make is supported by a statement that is accepted as true. There are three types of proofs (Paragraph, 3-Column, and Flow) We will only do 3-column proofs in this class.

To start, we will practice algebraic proofs….

You learned some properties in Algebra 1:

Properties of Real Numbers

Reflexive Property:

Symmetric Property:

Transitive Property:

Addition and Subtraction Properties:

Multiplication and Division Properties:

Substitution Property:

Distributive Property:

Ex2 : State the property that justifies each statement

a) If $\frac{x}{2} = 7$, then x = 14 ______________________

b) If x = 5 and b = 5, then x = b ______________________

c) If XY – AB = WZ – AB, then XY = WZ ______________________

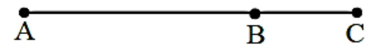
Example 3: Solve 3(x – 2) = 42. Justify each step.

We use a format similar to above, called a **two column proof (formal proof),** in geometry. These contain statements (left column) and reasons (right column).

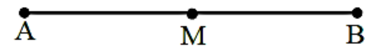
Example 2: Write a two column proof to show that $\frac{7d+3}{4} = 6$, then d = 3.

2-7 Proving Segment Relationships:

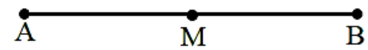
Segment Addition Postulate:
If B is between A and C, then ________________________.



Definition of Midpoint:
If M is the midpoint of $\overline{AB}$, then ________________________.



Midpoint Theorem
If M is the midpoint of $\overline{AB}$, then ___________________________.



Definition of Congruency
If AB = XY, then ________________________.
If $\overline{AB} \cong \overline{XY}$, then __________________________.

Definition of Segment Bisector
If $\overline{AB}$ bisects $\overline{XY}$, then ________________________________.

Reflexive Property	**Symmetric Property**	**Transitive Property**
AB = AB	If AB = CD, then CD = AB	If AB = CD and CD = EF, then AB = EF
$\overline{AB} \cong \overline{AB}$	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$

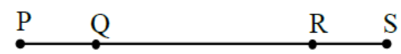
Ex 1) Given: Z is the midpoint of XY, XZ = 4x +1, and ZY = 6x – 13
Prove: x = 7.

Ex2) Given: AB = 5x + 2, BC = 3x – 10, AC = 10x - 16

Prove: x = 4

Ex3) Given: $\overline{PR} \cong \overline{QS}$.

Prove: $\overline{PQ} \cong \overline{RS}$.



Statements	**Reasons**
1)	1) Given
2)	2) Definition of Congruence
3) PQ + QR = PR ____ + ____ = _____	3)
4)	4) Substitution
5) QR = QR	
6)	5) Subtraction Property
7) $\overline{PQ} \cong \overline{RS}$.	7)

Ex4) Prove the following

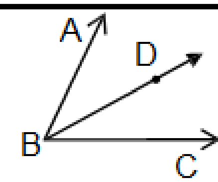
Given: $\overline{JK} \cong \overline{KL}, \overline{HJ} \cong \overline{GH}, \overline{KL} \cong \overline{HJ}$

Prove: $\overline{GH} \cong \overline{JK}$

Statements	**Reasons**
1) $\overline{JK} \cong \overline{KL}$, $\overline{KL} \cong \overline{HJ}$	1)
2)	2) Transitive Property
3) $\overline{HJ} \cong \overline{GH}$	3)
4)	4)
5)	5) Symmetric Property

Lesson 2.8: Proving Angle Relationships

Def. of an Angle Bisector: If *BD* bisects ∠ABC, then ______≅ ______



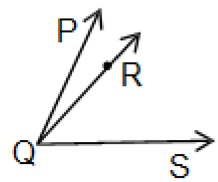
Def. of Supplementary Angles: If ∠X and ∠Y are supplementary, then _____ + ______ = _____

Def. of Complementary Angles: If ∠X and ∠Y are complementary, then _____ + ____ = ______

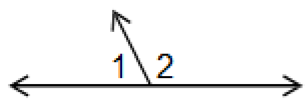
Def. of a Right Angle: If ∠K is a right angle, then ________ = _______

Def. of Congruency: If ∠P ≅ ∠D, then _____ = _____

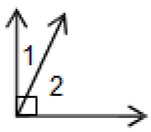
Angle Addition Postulate: If R is in the interior of ∠PQS, then ____ + ____ = ____



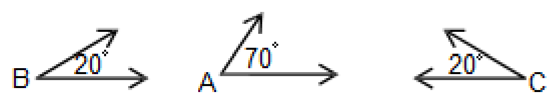
Supplement Theorem: If ∠1 and ∠2 form a linear pair, then they are ________________.



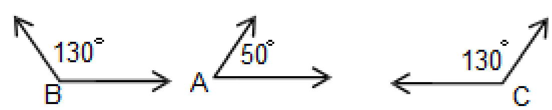
Complement Theorem: If ∠1 and ∠2 are adjacent and together they form a right angle, then they are ________________.



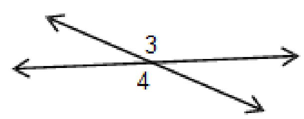
Congruent Supplements Theorem: If ∠B and C are both <u>supplementary</u> to ∠A, then ______ ≅ _______



Congruent Complements Theorem: If ∠B and C are both <u>complementary</u> to ∠A, then ______ ≅ _______



Vertical Angles Theorem: If ∠3 and ∠4 are vertical, ____ ≅ _____



<u>Perpendicular Lines and Right Angles</u>

Definition of perpendicular lines: If two lines are perpendicular, then they form right angles.

Theorem: All right angles are congruent.

Theorem: Perpendicular lines form congruent adjacent angles.

Theorem: If 2 angles are congruent and supplementary, then they are both right angles.

Theorem: If two congruent angles form a linear pair, then they are both right angles.

Reflexive Property	Symmetric Property	Transitive Property
m∠A = m∠A	If m∠A = m∠B, then m∠B = m∠A	If m∠A = m∠B and m∠B = m∠C, then m∠A = m∠C
∠A ≅ ∠A	If ∠A ≅ ∠B, then ∠B ≅ ∠A	If ∠A ≅ ∠B and ∠B ≅ ∠C, then ∠A ≅ ∠C

Example 1: **Given**: R in the interior of ∠PQS;
m∠PQS = 70°; m∠PQR = (14x – 44)°; m∠RQS = 5x°
Prove: x = 6

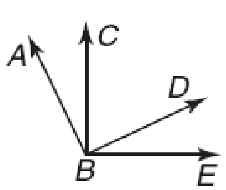
Sketch:

Statement	Answer	Reason
1. R in the interior of ∠PQS; m∠PQS = 70°; m∠PQR = (14x – 44)°; m∠RQS = 5x°		A. Substitution
2. m∠PQR + m∠RQS = m∠PQS		B. Simplify
3. (14x – 44) + 5x = 70		C. Division Prop.
4. 19x – 44 = 70		D. Given
5. 19x = 119		E. Addition Prop.
6. x = 6		F. Angle Addition Postulate

Example 2: **Given**: ∠O and ∠K are supplementary **Prove**: x =
25 m∠O = (4x + 10)°; m∠K = (3x – 5)°

Statement	Reason
1. ∠O and ∠K are supplementary m∠O = (4x + 10)°; m∠K = (3x – 5)°	1.
2. m∠O + m∠K = 180°	2.
3. (4x + 10) + (3x – 5) = 180	3.
4. 7x + 5 = 180	4.
5. 7x = 175	5.
6. x = 25	6.

Example 3: **Given**: ∠ABC and ∠CBD are complementary
∠DBE and ∠CBD form a right angle **Prove**: ∠ABC ≅ ∠DBE



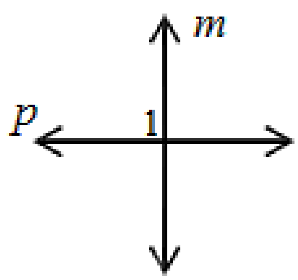
Statement	Reason
1.	1.
2. ∠DBE and ∠CBD are complementary	2.
3.	3.

Example 4: **Given:** $\overrightarrow{AT}$ bisects ∠SAX;
m∠SAT = (6x – 4); m∠TAX = (2x + 28) **Prove:** x = 8

Sketch:

Statement	Reason
1.	1.
2.	2. Definition of an angle bisector
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Example 5: **Given:** *p* ⊥ *m* **Prove:** x = 16
m∠1 = (4x + 26)°



Statement	Reason
1. *p* ⊥ *m*	1. Given
2. ______ is a right angle	2.
3. m∠1 = ______	3.
4.	4.
5.	5.
6.	6.