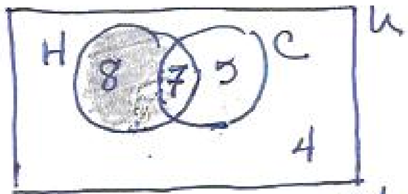
IB Math Studies
Topic 3 Test Review: Sets, Logic & Probability

Name: ______________________
Date: ______________________ Block: ______

1. All students at a party were asked whether they liked hamburgers or chicken. Eight students said they liked hamburgers only (H), 5 liked chicken only (C), 7 liked both, and 4 liked neither.

   (a) Draw a Venn diagram to represent this information, labeling the sets and writing the correct numbers in the region of the diagram.



   (b) Determine how many students were at the party. 24

   (c) Shade the region represented by $H \cap C'$.

   (d) Find the number of students who:

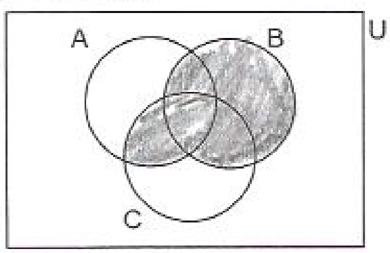
   (i) like hamburgers 15

   (ii) like chicken 12

   (iii) like hamburgers or chicken 20

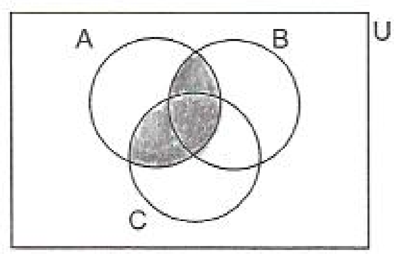
2. Shade the required region.

   (a)



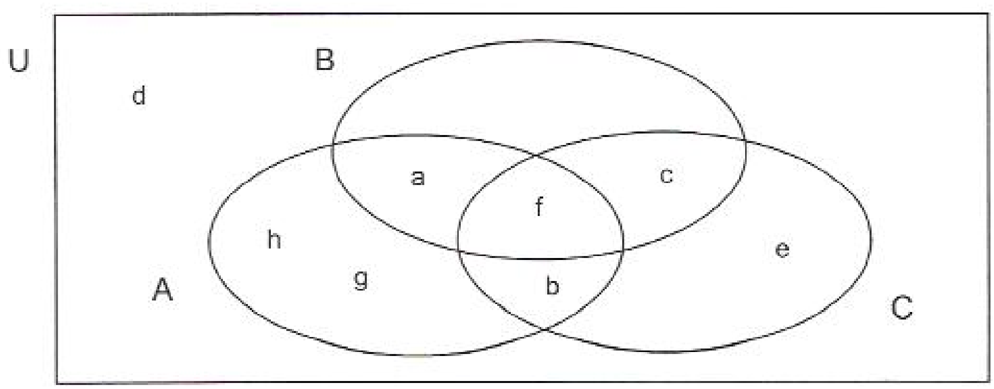
$(A \cap C) \cup B$

   (b)



$A \cap (B \cup C)$

3. The Venn diagram below, A, B, and C are subsets of a universal set U = {a, b, c, d, e, f, g, h}.



List the elements in each of the following sets.

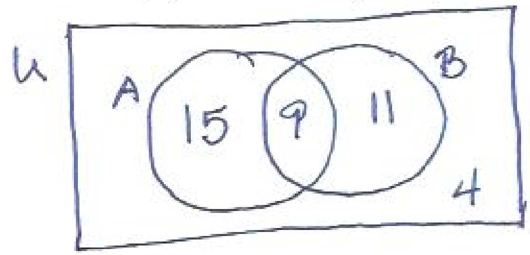
(a) $A \cup B$ a, b, e, f, g, h

(b) $A \cap B \cap C$ f

(c) $(A' \cap C) \cup B$ a, c, e, f

4. Draw a Venn diagram to show the number of people in each region if:

$n(U) = 39$ $n(A) = 24$ $n(B) = 20$ $n(A \cap B) = 9$



5. The sets U, O, A, and E are defined as follows:.

U = {all triangles}

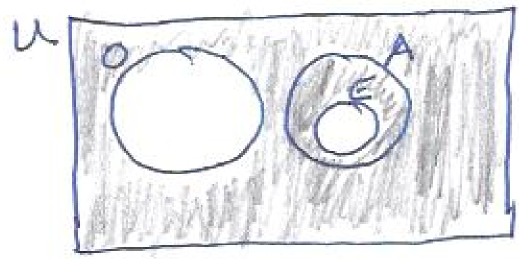
O = {all obtuse triangles}

A = {all acute triangles}

E = {all equilateral triangles}

all equilateral Δ's are also acute

(a) Draw a Venn diagram illustrating the relationship of these sets.



(b) Indicate $(O \cup E)'$ by shading the above diagram.

6. Consider each of the following statements:
p: Thomas is from Columbia **q**: Thomas is an architect **r**: Thomas plays the guitar

(a) Write each of the following in symbols:

(i) If Thomas is not an architect then he is not from Columbia. $\neg q \Rightarrow \neg p$

(ii) If Thomas is an architect, then he is either from Columbia or plays the guitar.
$q \Rightarrow (p \vee r)$

(b) Write the following argument in words: $\neg q \Rightarrow \neg(r \wedge p)$
If Thomas is not an architect, then he does not play the guitar and come from Columbia.

(c) Construct a truth table for the argument in part (b) using the values below for p, q, r and $\neg r$. Test whether the argument is logically valid.

p	q	r	$\neg q$	$r \wedge p$	$\neg(r \wedge p)$	$\neg q \Rightarrow \neg(r \wedge p)$
T	T	T	F	T	F	T
T	T	F	F	F	T	T
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Is the argument valid? YES NO

7. Let U = {x: $1 < x \leq 13$, $x \in \mathbb{N}$ }

P, Q, and R are subsets of U such that:

P = {odd numbers} 3, 5, 7, 9, 11, 13
Q = {multiples of 3} 3, 6, 9, 12
R = {factors of 30} 2, 3, 5, 6, 10

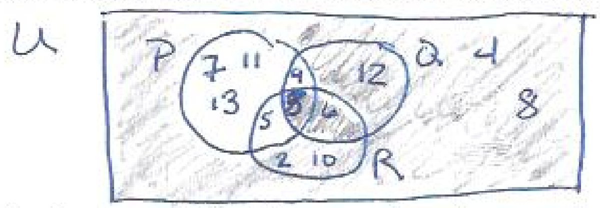
(a) List the elements of:

(i) U 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

(ii) $(P \cap Q) \cup R$ 2, 3, 5, 6, 9, 10

(b) Describe in words $Q \cap R$ Natural numbers greater than one and less than or equal to thirteen that are multiples of three and factors of thirty.

(c) (i) Draw a Venn diagram to show the relationship between sets P, Q, and R.



(ii) Write the elements of U in the appropriate places on the Venn diagram.

(d) Let p, q, and r be the statements:

p: x is an odd number
q: x is a multiple of 3
r: x is a factor of 30

(i) Write a sentence in words for the statement: $(q \wedge r) \vee \neg p$

x is a multiple of 3 and a factor of 30 or x is not an odd number

(ii) Shade the region on your Venn diagram in part (c)(i) that represents the truth set: $(q \wedge r) \vee \neg p$

(e) Use a truth table to determine the values of $(q \wedge r) \vee \neg p$.

p	q	r	$q \wedge r$	$\neg p$	$(q \wedge r) \vee \neg p$
T	T	T	T	F	T
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

(e) Write down all possible values of x for which $(q \wedge r) \vee \neg p$ is true.

2, 3, 4, 6, 8, 10, 12,

8. A box containing small tiles numbered 1 to 10 has one of the tiles randomly selected. What is the probability that the number selected is:

   (a) even? $5/10$ 2, 4, 6, 8, 10

   (b) greater than 3? $7/10$ 4, 5, 6, 7, 8, 9, 10

   (c) a prime number? $4/10$ 2, 3, 5, 7

   (d) a prime number and at least 5 ? $2/10$

9. A coin is tossed three times.

   (a) Write down all possible outcomes using *H* for heads and *T* for tails.

   H H H
   H H T
   H T H
   H T T
   T H H
   T H T
   T T H
   T T T

   (b) Write, as a fraction, the probability that

   (i) heads were the outcomes of all three tosses; $1/8$

   (ii) the outcomes of the first and second tosses were different $4/8$

10. A bag contains 6 red apples and 10 green apples. Without looking into the bag, Lena randomly selects one apple.

   (a) What is the probability that it is red? $6/16$

   The apple is red and Lena eats it (no replacement). Next the bag is passed to Jaime. Without looking into the bag, he randomly selects one apple.

   (b) What is the probability that it is green? $10/15$

   The apple is green and Jaime replaces it in the bag. Next the bag is passed José. Without looking in the bag, he randomly selects two apples.

   (c) What is the probability that they are both red? $\frac{20}{210}$ $\frac{5}{15} \cdot \frac{4}{14}$

11. The frequency distribution for how long (in minutes) customers stayed at a bistro is shown:

Time customer stayed in bistro (minutes)	Frequency (# of customers)
1-10	11
11-20	8
21-30	9
31-40	7
41-50	13
51-60	2

50 total

(a) What is the probability that a customer stayed between 11-20 minutes? $8/50$

(b) What is the probability that a customer stayed no more than 40 minutes? $35/50$ $11+8+9+7=35$

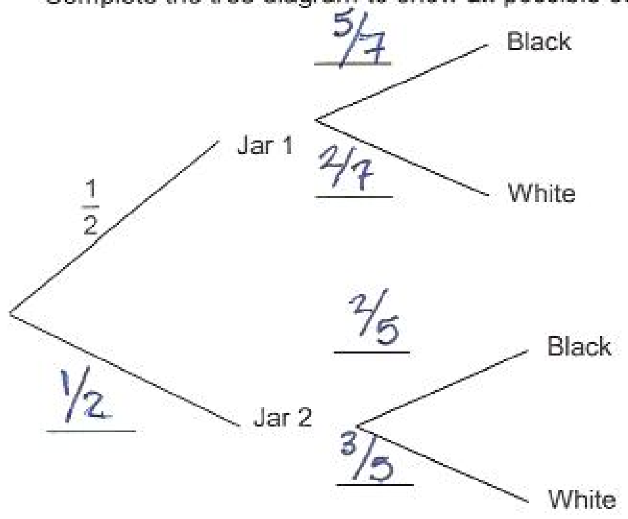
(c) What is the probability that a customer stayed between 21-50 minutes? $29/50$ $9+7+13$

11. (i) Two jars contain a number of coloured balls as indicated in the diagrams below.

Jar One **Jar Two**

First Experiment: A jar is first chosen at random and then a ball is drawn from that jar.

(a) Complete the tree diagram to show **all** possible outcomes of this experiment.



$P(J_1 \cap W) + P(J_2 \cap W)$

(b) What is the probability that a white ball is drawn? $\frac{31}{70}$ $\quad \frac{1}{2}\cdot\frac{2}{7} + \frac{1}{2}\cdot\frac{3}{5} =$

Second Experiment: The ball drawn in the first experiment is not replaced. A second ball is then drawn from the same jar.

(c) What is the probability that both balls are white? $\frac{73}{420}$ $\quad \frac{1}{2}\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)+\frac{1}{2}\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)$

12. Let F be the set of all families that have exactly two pets. Assuming P(cat) = P(dog) = 0.5 and one pet was adopted prior to the next pet:

(a) Write down the sample space for this situation. Use C for cat and D for dog.
$\Omega = \{CC, CD, DC, DD\}$

(b) Write, as a fraction, the probability that a family chosen at random has exactly

(i) two cats $\frac{1}{4}$

(ii) two cats, if it is already known (given) that the first pet is a cat $\frac{1}{2}$

(iii) two cats, if it is known that there is a cat in the family. $\frac{1}{3}$