

- In statistics, what is meant by the P-value? **The P-value is the probability of observing a value as extreme as the test statistic value by random chance given that the null hypothesis is true. The smaller the P-value, the stronger the evidence against  $H_0$  provided by the data.**
- Explain the difference between a *one-sided alternative hypothesis* and a *two-sided alternative hypothesis*. **In a one-sided alternative hypothesis, we make a prediction about the direction of the relationship (greater than the null hypothesized value OR less than the null hypothesized value). In a two-sided alternative hypothesis we do not predict a direction (“not equal to” the null hypothesized value). The p-value of a two-sided alternative will be twice that of a one-sided alternative.**
- What is meant by a *significance level*? **The probability of rejecting the null hypothesized value if the null is true (the alpha level, probability of a type I error). Also the “maximum” p-value we allow for rejection of the null-hypothesis.**
- What two circumstances guide in choosing a level of significance? **(1) How plausible is  $H_0$ ? (2) What are the consequences of rejecting  $H_0$ ? \*\*Also—do we want to avoid a Type I or Type II error?\***
- Why is it important to always plot your data? **We need to check for outliers which can cause misleading conclusions!**
- What is a *Type I Error*? **Incorrectly rejecting the null hypothesis. (Rejecting the null hypothesis when the null was true)**
- What is a *Type II Error*? **Incorrectly failing to reject the null hypothesis. (Failing to reject the null hypothesis when the null hypothesis was not true)**
- What is the relationship between the *significance level  $\alpha$*  and the probability of *Type I Error*? **Our significance level  $\alpha$  is the probability of a Type I Error.**
- What is meant by the power of a significance test? **The power of a significance test is the probability of correctly rejecting the null hypothesis.**
- Fill out the following chart with the following: Incorrect Decision, Correct Decision, Type I Error, Type II Error, Power,  $\alpha$ ,  $1 - \alpha$ ,  $\beta$ ,  $1 - \beta$

	$H_0$ is true	$H_a$ is true ( $H_0$ false)
Reject $H_0$	Incorrect Decision <b>Type I Error</b> $\alpha$	Correct Decision <b>POWER</b> $1 - \beta$
Fail to reject $H_0$	Correct Decision $1 - \alpha$	Incorrect Decision <b>Type II Error</b> $\beta$

- State four ways to increase the power of a significance test: **Increase  $\alpha$ , Consider a particular alternative that is farther away from  $\mu_0$ , Increase the sample size, Decrease  $\sigma$  by narrowing your focus to a subgroup of the population.**

### State the null and alternative hypotheses

12. A manufacturer claims that a new brand of air-conditioning unit uses only 6.5 kilowatts of electricity per day with a standard deviation of 1.4. A consumer agency believes the true figure is higher and runs a test on a sample of size 50. The sample mean is 7.0 kilowatts.

$$H_0: \mu = 6.5$$

$$H_a: \mu > 6.5$$

13. A local chamber of commerce claims that the mean family income level in a city is \$12,250. An economist believes that this is incorrect. He runs a hypothesis test, using a sample of 135 families, and finds a mean of \$11,500 with a standard deviation of \$3,180.

$$H_0: \mu = 12,250$$

$$H_a: \mu \neq 12,250$$

14. For each  $\alpha$  and observed significance level (p-value) pair, indicate whether the null hypothesis would be rejected.

- |                             |        |
|-----------------------------|--------|
| a) $\alpha = .05, p = .10$  | FTR    |
| b) $\alpha = .10, p = .05$  | Reject |
| c) $\alpha = .01, p = .001$ | Reject |
| d) $\alpha = .025, p = .05$ | FTR    |
| e) $\alpha = .10, p = .45$  | FTR    |

15. The one-sample Z statistic for testing

$$H_0: \mu = 0$$

$$H_a: \mu > 0$$

has the value  $Z = 1.98$ .

- a) Is the value  $Z = 1.98$  significant at the 5% level? **YES** Why? **P-value (0.0238) < Alpha level (0.05)**  
b) Is the value  $Z = 1.98$  significant at the 1% level? **NO** Why? **P-value (0.0238) > Alpha level (0.01)**

16. The one-sample Z statistic for the two-sided test of

$$H_0: \mu = 0$$

$$H_a: \mu \neq 0$$

has the value  $Z = -2.58$ .

- c) Is the value  $Z = -2.58$  significant at the 5% level? **YES... P-value (0.00988) < Alpha level (0.05)**  
a) Is the value  $Z = -2.58$  significant at the 1% level? **YES... P-value (0.00988) < Alpha level (0.01)**

17. City Emergency response times to serious accidents are reported at 6.7 minutes. Local officials are concerned that the actual response times are larger than the report indicates.

$$H_0: \mu = 6.7 \text{ minutes}$$

$$H_a: \mu > 6.7 \text{ minutes}$$

Describe and give the consequences of a Type I error and a Type II error. Decide whether you would choose an alpha level of 0.01 or 0.1 and explain **why**.

**Type I Error:** Local officials incorrectly reject the null hypothesis, claiming that the response times are **longer** than 6.7 minutes when they are **not**. The local officials may spend unnecessary money to make changes to decrease response time when this money could be used to fix real problems (or emergency responders get in trouble when they are actually making it within the expected time, etc.)

**Type II Error:** Local officials incorrect fail to reject the null hypothesis, claiming that the response times are **not longer** than 6.7 minutes when they actually **are**. Necessary changes will not be made in order to fix this problem and the emergency responders will continue to take too long to arrive at the scene of serious accidents, possibly resulting in an increased number of fatalities.

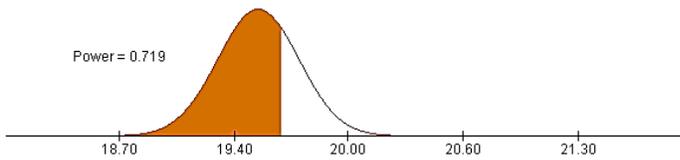
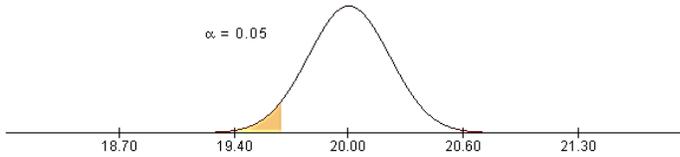
**I would choose an alpha level of 0.1 because I believe that Type II Error is more dangerous than a Type I error. A larger alpha level but will reduce the probability of a Type II Error.**

**...OR....**

**I would choose an alpha level of 0.01 because I believe that a Type I error is more dangerous than a Type II error. A smaller alpha level will make a Type I error less likely.**

18. State the probability of a Type I error and the probability of a Type II error.

$H_0: \mu = 20$    
   $H_a: \mu > 20$    
   $H_a: \mu < 20$    
   $H_a: \mu \neq 20$    
  $n = 20$    
 $\alpha = 0.05$    
 $\sigma = 1$    
 alt.  $\mu = 19.5$



**P(Type I Error) = 0.05, P(Type II Error) = 1 - Power = 1 - 0.719 = 0.281**

**Perform a FULL TEST for the following**

19. A professional pickologist believes that the average times a day a person picks their nose is 3, with a standard deviation of 1 nose pick. A researcher tests this claim to see if the true average is greater than 3, using an SRS of 500 people. She finds the sample mean to be 3.1 nose picks. Is this sufficient evidence at the 5% level to reject the professional pickologist's claim?

**STATE:** We want to estimate  $\mu$ , the true mean number of nose-picks per day for all people

$$H_0: \mu = 3$$

$$H_a: \mu > 3$$

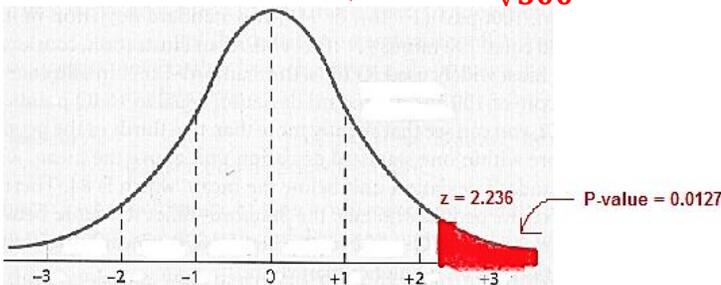
**PLAN:** Conduct a one-sample z-test for means ( $\sigma$  known)

1) SRS: We are told that the researcher took a simple random sample

2) Normality: We are not told if the population distribution is Normal. However, the **sample size of 500** is **large enough** to assume that the sampling distribution is **approximately Normal** due to the **Central Limit Theorem**.

3) Independence: The researcher sampled without replacement, but we can assume that there are at least  $10(500) = 5000$  members of the population. Also assume that the number of nose picks for each person in our SRS is independent of the other subjects.

$$\text{CALCULATE: } z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{3.1 - 3}{\frac{1}{\sqrt{500}}} = 2.236$$



**INTERPRET:** Because our p-value of 0.0127 is less than our significance level of 0.05, we can reject our null hypothesis. We have evidence that the true mean number of nose picks per day is greater than 3. (Although our results are statistically significant, they may not be practically significant. A difference of 0.1 picks is not very much!)

20. A pharmaceutical manufacturer does a chemical analysis to check the potency of toe fungus cream. The standard release potency for the active ingredient, fungicitis-eliminatus, is 20, with a known population deviation of 3. A sample of 20 lots gives the following potency data:

22	20	18	14	13	26	28	18	25	21
18	16	15	23	28	26	27	21	20	25

Is there significant evidence at the 5% level that the mean potency is not equal to the standard release potency? State your hypotheses, the Z statistic, the P-value, and report your conclusion.

**STATE:** We want to estimate  $\mu$ , the true mean potency of fungicitis-eliminatus for all bottles of this toe fungus cream.

$$H_0: \mu = 20$$

$$H_a: \mu \neq 20$$

**PLAN:** Conduct a one-sample z-test for means ( $\sigma$  known)

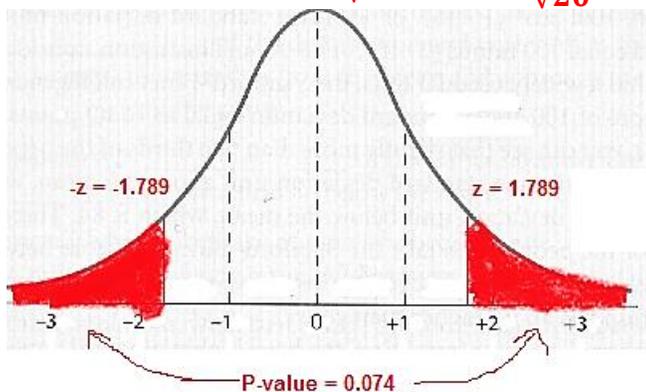
1) SRS: Assume that the pharmaceutical manufacturer took a simple random sample of creams.

2) Normality: Our sample size of 20 is not large enough to use the Central Limit Theorem and we were not told about the shape of the population; however, the boxplot only shows very minor skew so we can assume that the sampling distribution is approximately Normal.



3) Independence: The manufacturer samples without replacement, but we can assume that there are at least  $10(20) = 200$  total bottles of the toe fungus cream produced by the manufacturer. Also assume that the potency of each bottle of fungus cream is independent of the potency of other bottles.

**CALCULATE:** 
$$z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{21.2 - 20}{\frac{3}{\sqrt{20}}} = 1.789$$



**INTERPERATE:** Because our p-value of 0.074 is greater than our significance level of 0.05, we fail to reject our null hypothesis. We do not have evidence that the true mean potency of fungicitis-eliminatus is different from the standard release potency.