IB MATH STUDIES EXAM REVIEW: Calculus

1. Given the functions $f(x) = \frac{1}{4}x^2 - 2$ and $g(x) = x$.

(a) Differentiate $f(x)$ with respect to x.
(b) Differentiate $g(x)$ with respect to x.
(c) Calculate the value of x for which the gradients of the two graphs are the same.
(d) Draw the tangent to the parabola at the point with the value of x found in part (c).

2. The curve $y = px^2 + qx - 4$ passes through the point (2, –10).
(a) Use the above information to write down an equation in p and q.
The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1.
(b) (i) Find $\frac{dy}{dx}$.
(ii) Hence, find a second equation in p and q.
(c) Solve the equations to find the value of p and of q.

3. The table given below describes the behaviour of $f'(x)$, the derivative function of $f(x)$, in the domain $-4 < x < 2$.

x	$f'(x)$
$-4 < x < -2$	< 0
-2	0
$-2 < x < 1$	> 0
1	0
$1 < x < 2$	> 0

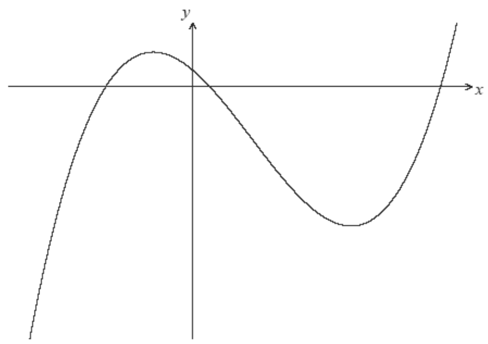
(a) State whether $f(0)$ is greater than, less than or equal to $f(-2)$. Give a reason for your answer.
The point P(–2, 3) lies on the graph of $f(x)$.
(b) Write down the equation of the tangent to the graph of $f(x)$ at the point P.
(c) From the information given about $f'(x)$, state whether the point (–2, 3) is a maximum, a minimum or neither. Give a reason for your answer.

4. Let $f(x) = 2x^2 + x - 6$
(a) Find $f'(x)$.
(b) Find the value of $f'(-3)$.
(c) Find the value of x for which $f'(x) = 0$.

5. Consider $f : x \mapsto x^2 - 4$.
(a) Find $f'(x)$.
Let L be the line with equation $y = 3x + 2$.
(b) Write down the gradient of a line parallel to L.
(c) Let P be a point on the curve of f. At P, the tangent to the curve is parallel to L. Find the coordinates of P.

6. The straight line, L, has equation $2y - 27x - 9 = 0$.
(a) Find the gradient of L.
Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to L.
(b) Write down $f'(x)$.
(c) (i) Find the x-coordinate of the point at which the tangent must be drawn.
(ii) Write down the value of $f(x)$ at this point.

7. The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.



(a) Write down the values of x where the graph of $f(x)$ intersects the x-axis.
(b) Write down $f'(x)$.
(c) Find the value of the local maximum of $y = f(x)$.
Let P be the point where the graph of $f(x)$ intersects the y-axis.
(d) Write down the coordinates of P.

(e) Find the gradient of the curve at P.

The line, L, is the tangent to the graph of $f(x)$ at P.

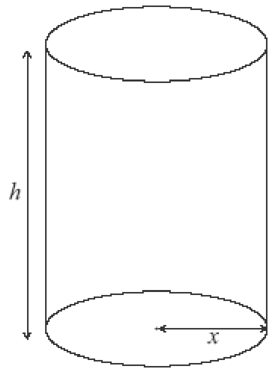
(f) Find the equation of L in the form $y = mx + c$.

There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L.

(g) Write down the gradient of the tangent at Q.

(h)Calculate the x-coordinate of Q.

8. A dog food manufacturer has to cut production costs by using as little aluminum as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm, and x represents the radius of the base of the can in cm.



The volume of the dog food cans is 600 cm^3.

(a) Show that $h = \dfrac{600}{\pi x^2}$.

(b) (i) Find an expression for the curved surface area of the can, in terms of x. Simplify your answer.

(ii) Hence write down an expression for A, the total surface area of the can, in terms of x.

(c) Differentiate A in terms of x.

(d) Find the value of x that makes A a minimum.

(e) Calculate the minimum total surface area of the dog food can.

9. Consider the function $f(x) = 3x + \dfrac{12}{x^2}$, $x \neq 0$.

(a) Differentiate $f(x)$ with respect to x.

(b) Calculate $f'(x)$ when $x = 1$.

(c) Use your answer to part (b) to decide whether $f(x)$ is increasing or decreasing at $x = 1$. Justify your answer.

(d) Solve the equation $f'(x) = 0$.

(e) The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P.

(i) Write down the coordinates of P.

(ii) Write down the gradient of T.

(iii) Write down the equation of T.

(f) Sketch the graph of the function f, for $-3 \leq x \leq 6$ and $-7 \leq y \leq 15$. Indicate clearly the point P and any intercepts of the curve with the axes.

(g) (i) On your graph draw and label the tangent T.

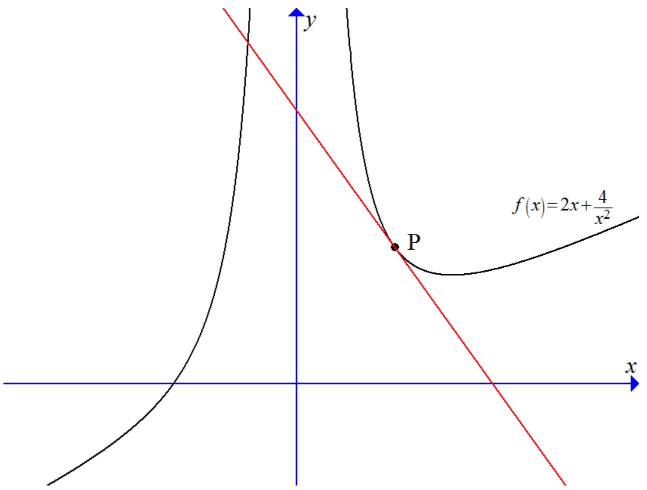
(ii) T intersects the graph of f at a second point. Write down the x-coordinate of this point of intersection.

10. Consider the function $y = x^2 - 7x - 44$.

(a) Find the equation of the normal at the point where $x = -3$.

(b) Find the coordinates of the point where the normal meets the curve again.

11. Consider the function $f(x) = 2x + \dfrac{4}{x^2}$.



(a) Find $f'(x)$.

(b) The tangent to $f(x)$ at point P has a gradient of -6. Find the coordinates of point P.

(c) Find the equation of the tangent at point P.

(d) Find where the tangent cuts the x-axis.

(e) Find the equation of the normal at P, expressing your answer in the form $y = mx + b$.

(f) Find the coordinates of both points at which the normal intersects $f(x)$ again.