

Practice Test

Find $f'(x)$.

1) $f(x) = 3x^4 - 6x^3 + 1$

2) $f(x) = \frac{3x^5 - 2x^4 + 3x^2 + 2x - 8}{x}$

3) $f(x) = 3x^2 + \frac{8}{x} - \frac{6}{x^3}$

4) (a) Differentiate the following function with respect to x :

$$f(x) = 2x - 9 - 25x^{-1}$$

(b) Calculate the x -coordinates of the points on the curve where the gradient of the tangent to the curve is equal to 6.

5) Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 5$.

(a) (i) Find $f'(x)$.

(ii) Find the gradient of the curve $f(x)$ when $x = 3$.

(b) Find the x -coordinates of the points on the curve where the gradient is equal to -12 .

(c) (i) Calculate the x -coordinates of the local maximum and minimum points.

(ii) Hence find the coordinates of the local minimum.

(d) For what values of x is the value of $f(x)$ increasing?

6) An open box has a square base of side x and height h .

a) Write down an expression for the volume, V , of the box.

b) Write down an expression for the total surface area, A , of the box.

The volume of the box is 1728 cm^3 .

c) Express h in terms of x .

d) Hence show that $A = 6912x^{-1} + x^2$.

e) Find $\frac{dA}{dx}$.

f) Calculate the value of x that gives a minimum surface area.

g) Find the surface area for this value of x .

7) Consider the function $f(x) = \frac{1}{2}x^3 - 2x^2 + 3$.

(a) Find $f'(x)$.

(b) Find $f''(x)$.

(c) Find the equation of the tangent to the curve of f at the point $(1, 1.5)$.

8) $f(x) = x^4 - 2x^2$

(a) Calculate $f''(x)$

(b) Determine the coordinates of any stationary points.

(c) Determine the nature of any stationary points

(d) Find where the graph intersects or touches:

i) the y -axis

ii) the x -axis

(e) Sketch the graph of $f(x)$

Practice Test

9) A stone is thrown vertically downwards off a tall cliff. The distance (s) it travels in metres is given by the formula $s = 4t + 5t^2$, where t is the time in seconds after the stone's release.

- (a) What is the rate of change of the distance with time $\frac{ds}{dt}$ (This represents the velocity.)
- (b) How many seconds after its release is the stone travelling at a velocity of 9 m s^{-1} ?
- (c) The stone hits the ground travelling at 34 m s^{-1} . How many seconds did the stone take to hit the ground?
- (d) Using your answer from part **c**, calculate the distance the stone falls and hence the height of the cliff.

10) The temperature ($T \text{ }^\circ\text{C}$) inside a pressure cooker is given by the formula $T = 20 + 12t^2 - t^3$; $t \leq 8$, where t is the time in minutes after the cooking started.

- (a) Calculate the temperature at the start.
- (b) What is the rate of temperature increase with time?
- (c) What is the rate of temperature increase when:
 - i. $t = 1$
 - ii. $t = 4$
 - iii. $t = 8$
- (d) The pressure cooker is turned off when $\frac{dT}{dt} = 36$ How long after the start could the pressure cooker have been switched off?
- (e) What was the temperature of the pressure cooker if it was switched off at the greater of the two times calculated in part **d**?

11) A function is given as $y = ax^2 + bx + 6$.

- a) Find $\frac{dy}{dx}$.
- b) The gradient of this function is 2 when x is 6. Write an equation in terms of a and b .
- c) The point $(3, -15)$ lies on the graph of the function. Find a second equation in terms of a and b .
- d) Use your GDC and your equations from parts b) and c) to find the values of a and b .

12) Given the following function: $f(x) = 3x^2 - 2x + 5$

- a) Calculate the equation of a tangent line passing through $x = 2$
- b) Calculate the equation of a tangent line passing through $x = -1$

13) $f(x) = (x - 2)^2 + 3$

- a) Calculate $f'(x)$
- b) Determine the range of values of x for which $f(x)$ is decreasing

14) Differentiate the following functions with respect to x . Then find the second derivative.

a) $f(x) = x(x + 2)$

b) $f(x) = (x + 2)(x - 3)$

c) $f(x) = \frac{x^3 - x}{x}$

d) $f(x) = \frac{1}{2x^2} + x$

15) What is the derivative of a function?

16) If the derivative of a function at a given point is positive, what does that mean?

17) If the derivative of a function at a given point is 0, what does that mean?