Lesson 6.3

Conditional Probability and Independence

- ✓ CALCULATE and INTERPRET conditional probabilities.
- ✓ USE the general multiplication rule to CALCULATE probabilities.
- ✓ USE tree diagrams to MODEL a chance process and CALCULATE probabilities involving two or more events.
- ✓ DETERMINE if two events are independent.
- ✓ When appropriate, USE the multiplication rule for independent events to COMPUTE probabilities.

What is Conditional Probability?

The probability we assign to an event can change if we know that some other event has occurred. This idea is the key to many applications of probability.

When we are trying to find the probability that one event will happen under the condition that some other event is already known to have occurred, we are trying to determine a conditional probability.

What is Conditional Probability?

The probability that one event happens given that another event is already known to have happened is called a **conditional probability**.

Suppose we know that event A has happened. Then the probability that event *B* happens given that event *A* has happened is denoted by $P(B \mid A)$.

prob. of B GIVEN A

Example 1: An AP Statistics class wants to know how common it is for teenagers to have their ears pierced. They collected data – gender and whether the student had a pierced ear – for 178 NPHS students. The two-way table below displays the data.

- Suppose we choose a student form the sample at random. Find the probability that a student

  a)has pierced ears

$$= \frac{103}{178}$$

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

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- Suppose we choose a student form the sample at random. Find the probability that a student
  - a)has pierced ears = 103/178
  - b)has pierced ears GIVEN that the student is male.

$$= \frac{19}{90} \qquad \frac{19/\cancel{178}}{90/\cancel{178}}$$

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
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Total		90	86	178

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Calculating Conditional Probabilities

To find the conditional probability $P(A \mid B)$, use the formula

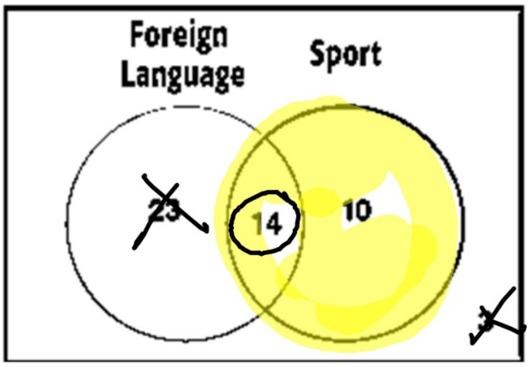
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

The conditional probability $P(B \mid A)$ is given by

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

Example 1: The data from a survey of 50 students is shown in the Venn Diagram. The students were asked whether or not they were taking a foreign language and whether or not they played a sport. The Venn diagram below displays the results. Event F = Foreign Language and Event S = Sport.

A student is selected at random, find the following probabilities



a) $P(F|S) = \dfrac{14}{24}$

$= \dfrac{P(F \cap S)}{P(S)}$

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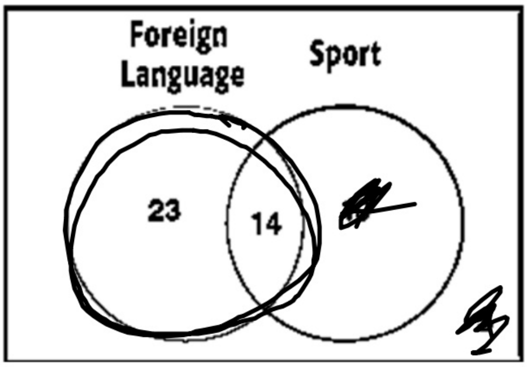

b) P(S|F) $= \frac{14}{37}$

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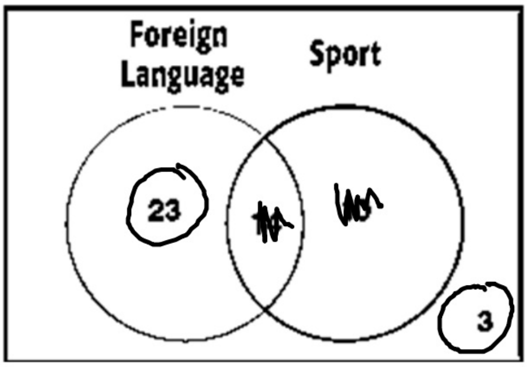
A student is selected at random, find the following probabilities

c) $P(F|S^c) = \frac{23}{26}$



The General Multiplication Rule

General Multiplication Rule

The probability that events A and B both occur can be found using the **general multiplication rule**

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

where $P(B \mid A)$ is the conditional probability that event B occurs given that event A has already occurred.

$P(\heartsuit) = \frac{13}{52}$

$P(\heartsuit \cap R) = \frac{13}{52} \cdot \frac{25}{51}$

$P(R) = \frac{26}{52}$

$P(R \mid \heartsuit) = \frac{25}{51}$

The General Multiplication Rule

Example: The Pew internet and American Life Project found that 93% of teens use the internet, and that 55% of online teens have posted a profile picture.

$P(P|I)$

Find the probability that a randomly selected teen uses the intenet and has posted a profile.

$P(I \cap P) = .93 \cdot 0.55 = 0.51$

General Multiplication Rule

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The General Multiplication Rule - Tree Diagrams!

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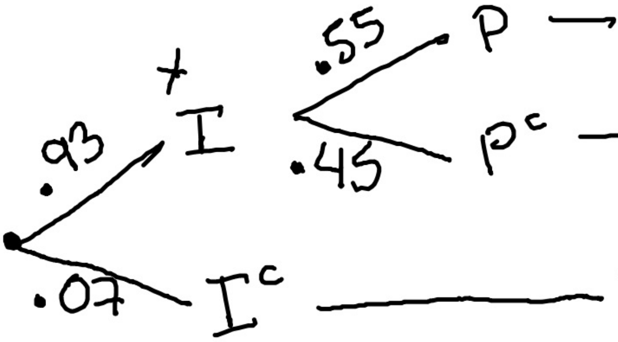

$P(I \cap P) = 0.51$

$P(I \cap P^c) = 0.42$

$I^c \cap P^c = 0.07$

Tree Diagrams!!!

Example: A videocassette recorder (VCR) manufacturer receives 70% of his parts from factory F1 and the rest from factory F2. Suppose 3% of the output from F1 are defective, while only 2% of the output from F2 are defective. What is the probability the part is defective?

Tree Diagrams!!!

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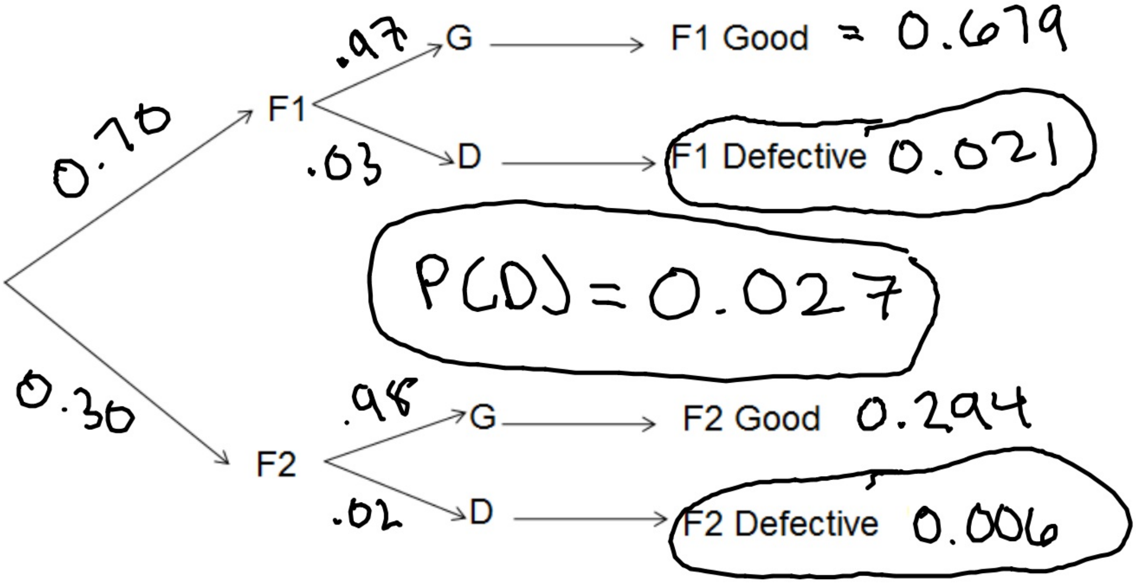
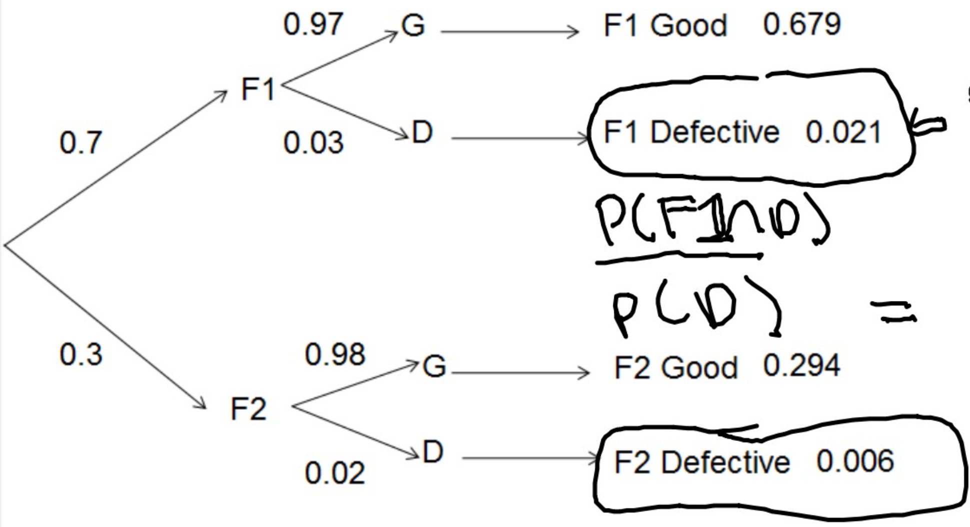


- **Given that a randomly chosen part is defective, what is the probability that it came from factory F1?**



$\leftarrow P(F1 \cap D)$

$$\frac{P(F1 \cap D)}{P(D)} = \frac{0.021}{0.027}$$

Conditional Probability and Independence

When knowledge that one event has happened does not change the likelihood that another event will happen, we say that the two events are **independent**.

Two events A and B are **independent** if the occurrence of one event does not change the probability that the other event will happen. In other words, events A and B are independent if

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B).$$

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When events A and B are independent, we can simplify the general multiplication rule since $P(B \mid A) = P(B)$.

Multiplication rule for independent events

If A and B are independent events, then the probability that A and B both occur is

$$P(A \cap B) = P(A) \cdot P(B)$$

Following the Space Shuttle Challenger disaster, it was determined that the failure of O-ring joints in the shuttle's booster rockets was to blame. Under cold conditions, it was estimated that the probability that an individual O-ring joint would function properly was 0.977.

Assuming O-ring joints succeed or fail independently, what is the probability all six would function properly?

$0.977 \times 0.977 \ldots$

$0.977^6 = 0.87$

Following the Space Shuttle Challenger disaster, it was determined that the failure of O-ring joints in the shuttle's booster rockets was to blame. Under cold conditions, it was estimated that the probability that an individual O-ring joint would function properly was 0.977.

Assuming O-ring joints succeed or fail independently, what is the probability at least one will not function properly?

$S = \{0, \boxed{1, 2, 3, 4, 5, 6}\}$

$1 - P(\text{all function properly})$

$1 - 0.87 = \boxed{0.13}$

$.977^6$

Independent vs. Dependent Events

- Two events A and B are independent if knowing that one occurs does not change the probability of that the other occurs.

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B).$$

Determining Independence

To check if events are independent, you can check for either of the following:

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B)$$

OR

$$P(A \cap B) = P(A) \cdot P(B)$$

Determining Independence

A B

Are the events "male" and "has pierced ears" independent?

Check both if first = yes

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B)$$

$$\frac{19}{103} \stackrel{?}{=} \frac{90}{178}$$

$.18 \neq 0.51$

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

Not independent

$P(A \mid B) \neq P(A)$

Determining Independence

Are the events "male" and "has pierced ears" independent?

A = male, B = has pierced ears

$$P(A \cap B) = P(A) \cdot P(B)$$

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

$$\frac{19}{178} \stackrel{?}{=} \frac{90}{178} \cdot \frac{103}{178}$$

$$0.11 \neq 0.29$$

Not ind.

$$P(A \cap B) \neq P(A) \cdot P(B)$$

Independent Events

- You flip a coin and roll a 6-sided die. What is the probability that you get tails **and** 3?

Mutually Exclusive

You roll a 6-sided die. What is the probability that you roll an odd number **or** a 4?