6-2 : Probability Basics

- The **sample space** is a list of all possible outcomes in a given situation.
- EX: You flip a coin. List the sample space.

S= {H, T}

- EX2: You have 3 red slips of paper numbered 1 – 3 and 2 blue slips of paper numbered 1 – 2. List the sample space.

S= {R1, R2, R3, B1, B2}

- A **probability model** is a description of some chance process that consists of two parts:
  1. The **sample space S** (the set of all possible outcomes)
  2. The probability for each outcome.

outcomes	H	T
prob.	0.5	0.5

- **Example #1:** You roll a single die. Give the probability model for this chance process.
- S = {1, 2, 3, 4, 5, 6}.
- Probability model:

Outcome	1	2	3	4	5	6
Probability	1/6	1/6	1/6	1/6	1/6	1/6

○ An **event** is any outcome or a set of outcomes of a random phenomenon.

- An event is a subset of the sample space
- Events are usually designated by capital letters (*A, B, C, etc.)*

- For the probability model below, we may define A = number less than 4. Find P(A).

Outcome	1	2	3	4	5	6
Probability	1/6	1/6	1/6	1/6	1/6	1/6

P(A) = 1/2

- B = number is 6. Find P(A or B) = 4/6

1) The probability *P(A)* of any event *A* satisfies $\boxed{0 \le P(A) \le 1}$

2) If *S* is the sample space in a probability model, then $\boxed{P(S) = 1}$

3) ***Complement rule*:** **$P(A^C) = 1 - P(A)$.**

- The *complement* of any event A is the event that A does not occur, written as A^C.

Addition Rule of Mutually Exclusive Events: If A and B are mutually exclusive, P(A or B) = P(A) + P(B)

- Two events A and B are ***Mutually Exclusive*** **(also called** disjoint **)** if they cannot occur @ the same time / no "events" in common
- In other words, $\boxed{P(A \text{ and } B) = 0}$.

Example: You draw one card from a standard deck. Event A = drawing a king and Event B = Drawing a 2.

The events are disjoint because: P(A and B) = 0

Find P(King or 2) = P(A∪B) = $\frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

"or"

Distance learning courses are rapidly gaining popularity among college students. Below is a probability model showing the proportion of all distance learners in each age group

Age Group (yr)	18 to 23	24 to 29	30 to 39	40 or over
Probability	.57	.17	.14	.12

P(18 to 23) = 0.57

P(at least 24) = 0.43 ← 0.17 + 0.14 + 0.12

1 − 0.57

- **The General Addition Rule:** P (A or B) = P(A)+P(B) − P(A∩B)
  - **Example:** What is the probability of drawing a red card or a king?

$$P(R \cup K) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$$

double-counted Red kings

Example 1: An AP Statistics class wants to know how common it is for teenagers to have their ears pierced. They collected data – gender and whether the student had a pierced ear – for 178 NPHS students. The two-way table below displays the data.

- Suppose we choose a student form the sample at random. Find the probability that a student
  - a) has pierced ears
  - b) is male and has pierced ears
  - c) is male or has pierced ears
  - d) is female or doesn't have pierced ears

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

- Suppose we choose a student form the sample at random. Find the probability that a student
  - a) has pierced ears

$$= \frac{103}{178}$$

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

- Suppose we choose a student form the sample at random. Find the probability that a student
  - b) is male and has pierced ears

$$= \frac{19}{178}$$

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

- Suppose we choose a student form the sample at random. Find the probability that a student

c) is male or has pierced ears

$= \frac{174}{178}$

		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

$\frac{90+103-19}{178}$

- Suppose we choose a student form the sample at random. Find the probability that a student

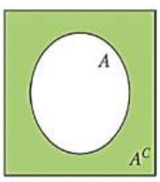
d) is female or doesn't have pierced ears

$= \frac{159}{178}$

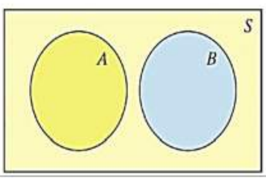
		Gender		
		Male	Female	Total
Pierced Ears?	Yes	19	84	103
	No	71	4	75
Total		90	86	178

Venn Diagrams!!!!!

The complement A^c contains exactly the outcomes that are not in A.



The events A and B are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.



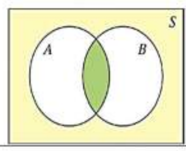
Venn Diagrams!!!!!

and = ∩

or = ∪

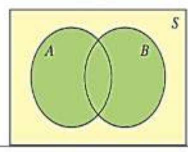
The intersection of events A and B ($A \cap B$) is the set of all outcomes in both events A and B.

$A \cap B$



The union of events A ~~and~~ OR B ($A \cup B$) is the set of all outcomes in either event A or B.

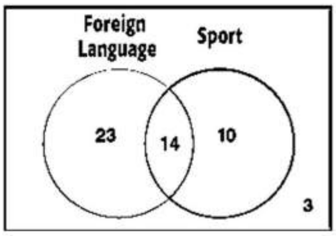
$A \cup B$



Venn Diagrams!!!!!

Example 1: The data from a survey of 50 students is shown in the Venn Diagram. The students were asked whether or not they were taking a foreign language and whether or not they played a sport. The Venn diagram below displays the results. Event F = Foreign Language and Event S = Sport.

- P(F) =
- P(S) =
- P(F ∪ S) =
- $P(S^c)$ =
- P(F ∩ S)



Venn Diagrams!!!!!

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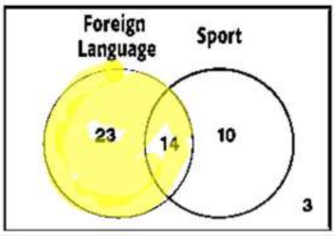
- P(F) = $\frac{37}{50}$



Venn Diagrams!!!!!

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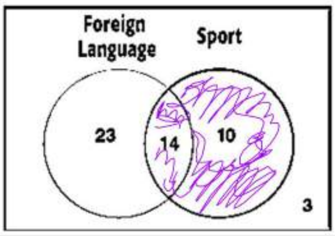
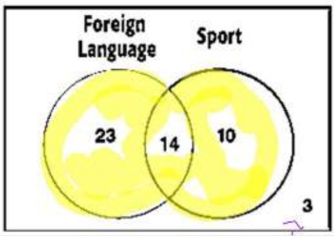
- P(S) = $\frac{24}{50}$



Venn Diagrams!!!!!

Example 1: The data from a survey of 50 students is shown in the Venn Diagram. The students were asked whether or not they were taking a foreign language and whether or not they played a sport. The Venn diagram below displays the results. Event F = Foreign Language and Event S = Sport.

- P(F ∪ S) = $\frac{47}{50}$

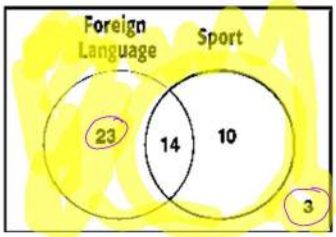


7

Venn Diagrams!!!!!

Example 1: The data from a survey of 50 students is shown in the Venn Diagram. The students were asked whether or not they were taking a foreign language and whether or not they played a sport. The Venn diagram below displays the results. Event F = Foreign Language and Event S = Sport.

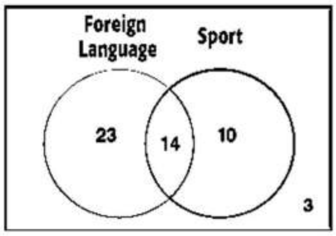
- $P(S^c) = \frac{26}{50}$



Venn Diagrams!!!!!

Example 1: The data from a survey of 50 students is shown in the Venn Diagram. The students were asked whether or not they were taking a foreign language and whether or not they played a sport. The Venn diagram below displays the results. Event F = Foreign Language and Event S = Sport.

- $P(F \cap S) = \frac{14}{50}$



- **Example 2:** Real estate ads in the North Pole claim that 64% of polar bear caves have only one entrance, 21% of caves contain hieroglyphics, and 17% have both features. What is the probability that a polar bear cave has…

  Make a Venn Diagram of the information
  Only one entrance **or** hieroglyphics?
  Neither only one entrance **nor** hieroglyphics?
  Only one entrance **and** no hieroglyphics?

Venn Diagrams!!!!!

- **Example 2:** Real estate ads in the North Pole claim that 64% of polar bear caves have only one entrance, 21% of caves contain hieroglyphics, and 17% have both features. What is the probability that a polar bear cave has…

  a) Only one entrance **or** hieroglyphics?