Free Response Communicate your thinking clearly and completely.

8. A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6%, with the additional revenue going to education. Let $\hat{p}$ denote the proportion in the sample who say they support the increase. Suppose that 40% of all adults in Ohio support the increase.

(a) If $\hat{p}$ is the proportion of the sample who support the increase, what is the mean of the sampling distribution of $\hat{p}$?

$\mu_{\hat{p}} = 0.4$

(b) What is the standard deviation of the sampling distribution of $\hat{p}$?

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(0.6)}{1500}} = \sigma_{\hat{p}} = 0.0126$

(c) Explain why you can use the formula for the standard deviation of $\hat{p}$ in this setting.

We can assume that there are at least $10(1500) = 15{,}000$ adults in Ohio so we can assume independence.

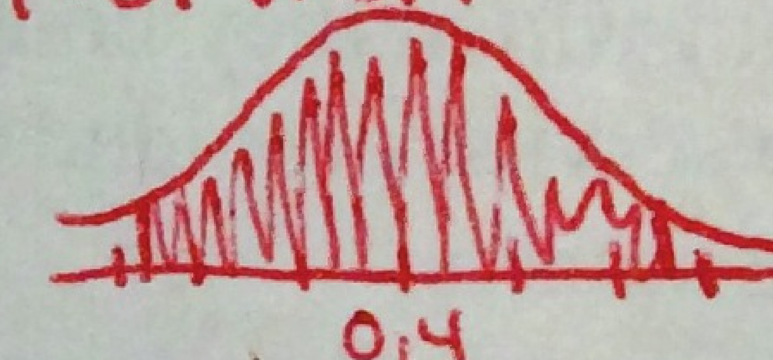
(d) Check that you can use the Normal approximation for the distribution of $\hat{p}$.

$np \geq 10$ $\quad n(1-p) \geq 10$

$1500(.4) \geq 10$ $\quad 1500(.6) \geq 10$ ✓ The sampling dist is Appx Normal

$600 \geq 10$ $\quad 900 \geq 10$

(e) Find the probability that $\hat{p}$ takes a value between 0.37 and 0.43.



$P(0.37 \leq \hat{p} \leq 0.43) = \text{ncdf}(0.37, 0.43, .4, .013) = 0.98$ → The probability that $\hat{p}$ takes a value between 0.37 & 0.43 is 98%.

(f) How large a sample would be needed to guarantee that the standard deviation of $\hat{p}$ is no more than 0.01? Explain.

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

$0.01 \geq \sqrt{\frac{.4(.6)}{n}}$

$\boxed{n = 2400}$

$\mu = 12 \quad \sigma = 0.4$

11. A certain beverage company is suspected of underfilling its cans of soft drink. The company advertises that its cans are normally distributed with an average volume pf 12 ounces and a standard deviation 0.4 ounce. For the questions that follow, suppose that the company is telling the truth.

(a) Can you calculate the probability that a single randomly selected can contains 11.9 ounces or less? If so, do it. If not, explain why you cannot. Yes... the pop is normally distributed.

$P(X \le 11.9) = \text{ncdf}(-1\times10^{99}, 11.9, 12, 0.4) = \boxed{0.40129}$ SENTENCE

11.6 12

(b) A quality control inspector measures the contents of an SRS of 50 cans of the company's soda and calculates the sample mean $\bar{x}$. What are the mean and standard deviation of the sampling distribution of $\bar{x}$ for samples of size $n = 50$?

$\mu_{\bar{x}} = \mu = 12 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.4}{\sqrt{50}} = 0.0566$

(c) The inspector in part (b) obtains a sample mean of $\bar{x} = 11.9$ ounces. Calculate the probability that a random sample of 50 cans produces a sample mean fill of 11.9 ounces or less.

$P(\bar{x} \le 11.9) = \text{ncdf}(-1\times10^{99}, 11.9, 12, 0.0566) = 0.0385$ The prob that a rand sample of 50 cans will have an $\bar{x}$ fill of 11.9 oz or less is 0.0385.

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(d) What would you conclude about whether the company is underfilling its cans of soda? Justify your answer. YES! There is only a 3.85% chance that an average fill of 11.9 oz or less will occur by chance in a sample of 50 cans if the population mean is actually $\mu = 12$ oz. Because this is so unlikely, we have evidence that the company is underfilling cans!

12. A hot dog manufacturer claims its most popular brand of hot dog has an average fat content of 18g per hot dog. Suppose the standard deviation of the fat content of all hot dogs is 1g and that the distribution of fat content is normally distributed. An independent testing organization selects an STS of 36 hot dogs and finds the average fat content is 18.4g. Does this result indicate that the manufacturer's claim is incorrect?

$\mu_{\bar{x}} = \mu = 18$ Assume the hot dog manufacturer produces at least $10(36) = 360$ hot dogs.

$\sigma = \sigma/\sqrt{n} = 1/\sqrt{36} = 0.1667$

BECAUSE our sample size of 36 is greater than $n = 30$, we can say that the sampling dist. is appx Normal due to the CLT.

$P(\bar{x} \ge 18.4) = \text{ncdf}(18.4, \infty, 18, 0.1667)$

$P(\bar{x} \ge 18.4) = 0.008$

13. The article "Thrillers" (Newsweek, Apr. 22, 1985) states "Surveys tell us that more than half of America's college graduates are avid readers of mystery novels." Assume the true proportion is exactly 0.5. What is the probability that an SRS of 225 college graduates would give a sample proportion greater than 0.6?

$p = 0.5 \rightarrow \mu_{\hat{p}} = 0.5$ Assume there are at least $(10)(225) = 2250$ college grads in the population $\rightarrow \sigma_{\hat{p}} = \sqrt{\frac{.5(1-.5)}{225}} = 0.0333$

$np \ge 10 \quad n(1-p) \ge 10$

$225(.5) \ge 10 \quad 225(.5) \ge 10$

$112.5 \ge 10 \quad 112.5 \ge 10$

The sampling dist. is Appx Normal ✓

.5

$P(\hat{p} \ge 0.6) = \text{ncdf}(.06, \infty, .5, .0333) = 0.0013$

The prob. that an SRS of 225 grads will have $\hat{p} = 0.0013$ who are avid readers of mysteries is .13%

14. Define the following distributions:

a) Population distribution - The dist. of all individuals in a population.

b) Distribution of the sample - The dist of individuals in a sample

c) Sampling distribution – The dist of all sample means ($\bar{x}$) or sample proportions ($\hat{p}$) for a specific sample size.

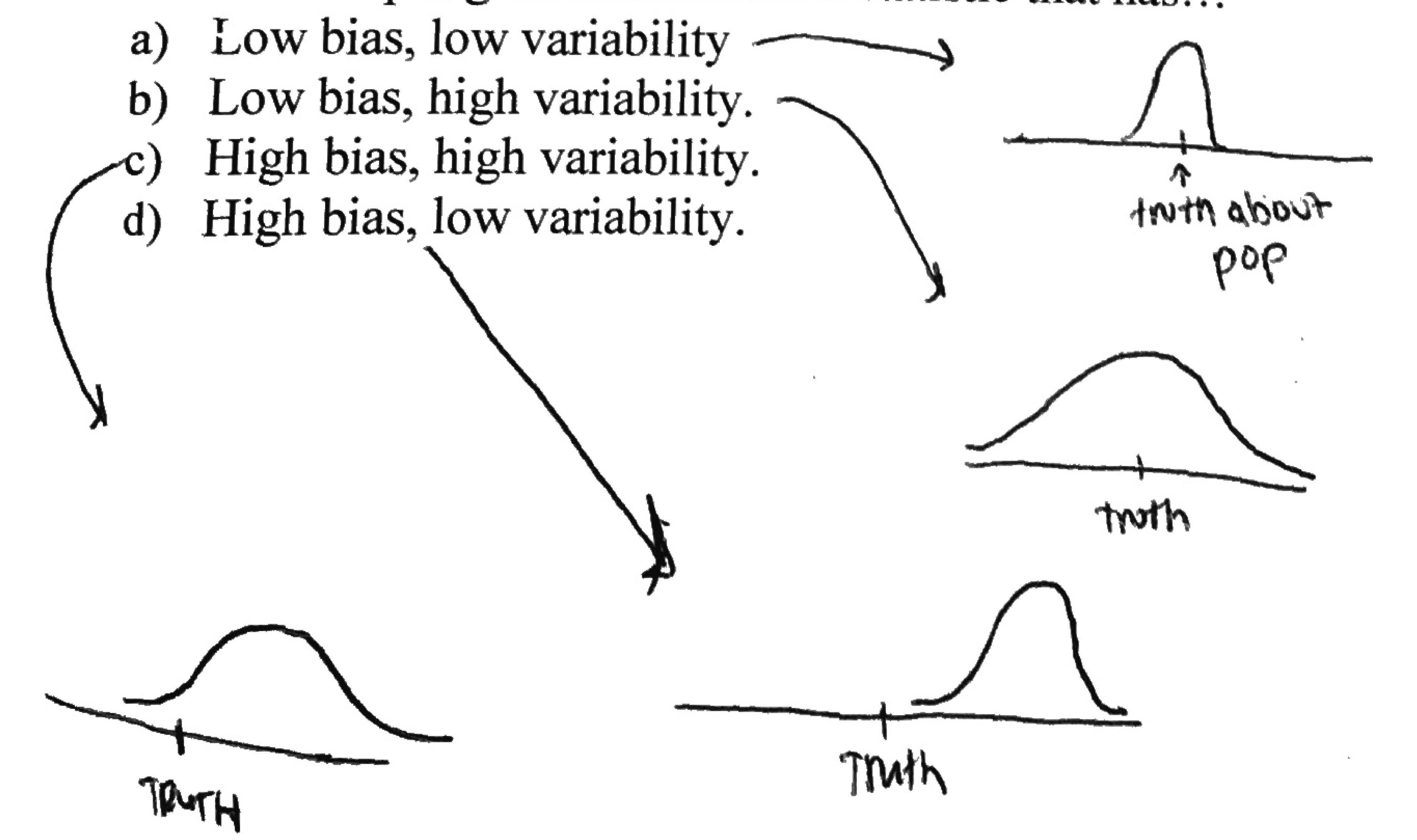
15. Draw a sampling distribution for a statistic that has…

a) Low bias, low variability

b) Low bias, high variability.

c) High bias, high variability.

d) High bias, low variability.



YES!

→ The probability that an SRS of 36 hot dogs would have a mean fat content of at least 18.4g if the true pop. mean, μ, is 18g is only 0.8%. Because this is so low, we have evidence that the manufacturer's claim is incorrect.