CHAPTER 12
Testing a Claim

12.2
Tests About a Population Proportion

Tests About a Population Proportion

- ✓ STATE and CHECK the Random, 10%, and Large Counts conditions for performing a significance test about a population proportion.
- ✓ PERFORM a significance test about a population proportion.
- ~~✓ INTERPRET the power of a test and DESCRIBE what factors affect the power of a test.~~

Carrying Out a Significance Test

Let's say that Ms. Keeler now claims to be an 80% free-throw shooter. In a sample 50 free-throws, she makes 32. Her sample proportion of made shots, 32/50 = 0.64, is much lower than what she claimed.

Does it provide *convincing* evidence against this claim?

① STATE

To find out, we must perform a significance test of

H_0: $p = 0.80$

H_a: $p < 0.80$

where p = true proportion of FT Keeler makes (in all attempts)

$\alpha = 0.05$

Carrying Out a Significance Test

H_0: p = 0.80

H_a: p < 0.80

Conditions For Performing A Significance Test About A Proportion

- **Random**: The data come from a well-designed random sample or randomized experiment.
- **10%**: When sampling without replacement, check that $n \le (1/10)N$.
- **Large Counts**: Both np_0 and $n(1 - p_0)$ are at least 10.

← null hyp. value

PLAN: one-sample z-test for proportions

1) Not a random sample (proceed w/ caution)

2) Assume Keeler shoots @ least 10(50) = 500 FT in a season.

3) $np_0 \ge 10$ $\quad n(1-p_0) \ge 10$

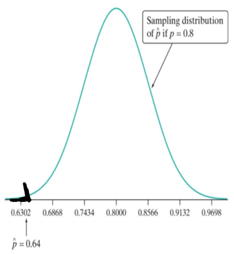
$50(.8) \ge 10$ $\quad 50(1-.8) \ge 10$

$40 \ge 10$ $\quad 10 \ge 10$

Carrying Out a Significance Test

If the null hypothesis $H_0: p = 0.80$ is true, then the player's sample proportion of made free throws in an SRS of 50 shots would vary according to an approximately Normal sampling distribution with mean

$$\mu_{\hat{p}} = p = 0.80 \text{ and standard deviation } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.8)(0.2)}{50}} = 0.0566$$



Carrying Out a Significance Test

A significance test compares the value of the parameter (true pop mean, as stated in the null) with the calculated sample mean. Values of the sample **far** from the true parameter give evidence **against H_0**

To assess how far the sample statistic is from the population parameter, we have to standardize it (to make comparison)

A **test statistic** measures how far a sample statistic diverges from what we would expect if the null hypothesis H_0 were true, **in standardized units**. That is,

$$\text{test statistic} = \frac{\text{statistic - parameter}}{\text{standard deviation of statistic}}$$

Carrying Out a Significance Test

In our free-throw shooter example, the sample proportion 0.64 is pretty far below the hypothesized value H_0: p = 0.80.

Standardizing, we get

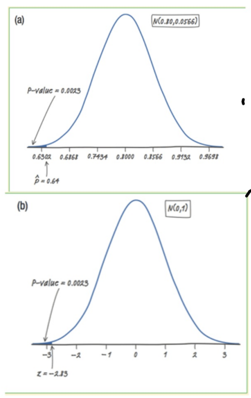
"DO"

$$\text{test statistic} = \frac{\text{statistic - parameter}}{\text{standard deviation of statistic}}$$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.64 - 0.80}{\sqrt{\frac{.80(1-.80)}{50}}}$$

$$z = -2.83$$

Carrying Out a Significance Test



Using Table A, we find that the P-value is $P(z \le -2.83) = 0.0023$.

$$\text{ncdf}(-1\times10^{99}, -2.83, 0, 1) = 0.0023$$

CONCLUDE: Because our p-value (0.0023) is less than our significance level ($\alpha = 0.05$) we REJECT Ho. We have evidence that Keeler's FT%age is less than 80%.

The One-Sample z-Test for a Proportion

Significance Tests: A Four-Step Process
State: What hypotheses do you want to test, and at what significance level? Define any parameters you use.
Plan: Choose the appropriate inference method. Check conditions.
Do: If the conditions are met, perform calculations.
• Compute the **test statistic**. ← formula → subst.
• Find the **P-value**. ← sketch!
Conclude: Make a decision about the hypotheses in the context of the problem.

Example#1: All together=)

Two-Sided Test

- Let's be honest, pink Starburst is so much better than the other flavors (I mean...why does orange even exist!?). Starburst claims that their colors are evenly distributed (25% of each). I want to know if the proportion of pink Starburst is really 0.25. I buy a bag of Starburst and find 31 pink Starburst out of 148 candies. Do I have evidence that the proportion of pink Starburst is **not** 25%?

STATE: $H_0: p = 0.25$ $H_A: p \neq 0.25$

$p =$ **The true proportion of pink starburst candies (in all starburst)**

$\alpha = 0.05$

Plan: **1) Assume that the sample was an SRS**
2) Assume that at least 10(148) = 1480 starburst in the population of all starburst candies
3) $np_0 \geq 10$ $n(1-p_0) \geq 10$
$148(.25) \geq 10$ $148(.75) \geq 10$
$37 \geq 10$ $111 \geq 10$

→ Two-Sided Test ←

$H_0: p = 0.25$ $H_A: p \neq 0.25$
$p < 0.25$ or $p > 0.25$

- Let's be honest, pink Starburst is so much better than the other flavors (I mean...why does orange even exist!?). Starburst claims that their colors are evenly distributed (25% of each). I want to know if the proportion of pink Starburst is really 0.25. I buy a bag of Starburst and find 31 pink Starburst out of 148 candies. Do I have evidence that the proportion of pink Starburst is **not** 25%?

DO: $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.209 - .25}{\sqrt{\frac{.25(.75)}{148}}} = -1.15$$

P-value $= 2\,[\text{ncdf}(-\infty, -1.15, 0, 1)]$

P-value $= 0.2501$

Because our p-value (0.2501) is more than our sign. level ($\alpha = 0.05$) we Fail To Reject H_0. We do NOT have evidence that the true prop. of pink SB differs from 0.25.

Two-Sided Tests

The P-value in a one-sided test is the area in one tail of a standard Normal distribution—the tail specified by Ha.

In a two-sided test, the alternative hypothesis has the form

$$H_a : p \neq p_0.$$

The P-value in such a test is the probability of getting a sample proportion as far as or farther from p_0 in **either direction** than the observed value of p-hat.

As a result, you have to find the area in both tails of a standard Normal distribution to get the P-value.

Why Confidence Intervals Give More Information

The result of a significance test is basically a decision to reject H_0 or fail to reject H_0. When we reject H_0, we're left wondering what the actual proportion p might be. A confidence interval might shed some light on this issue.

95% $\hat{p} = 0.209$ $n = 148$

Construct a confidence interval for the true proportion of pink Starburst candies.

(.143, .275)

Why Confidence Intervals Give More Information

There is a link between confidence intervals and two-sided tests.

The 95% confidence interval gives an approximate range of p_0's that would not be rejected by a two-sided test at the $\alpha = 0.05$ significance level.

✓ A two-sided test at significance level α (say, $\alpha = 0.05$) and a $100(1 - \alpha)$% confidence interval (a 95% confidence interval if $\alpha = 0.05$) give similar information about the population parameter.

HW# 2: 12.23, 12.24*, 12.29*, 11.23