12.1 Homework for "t" Hypothesis Tests

1) Below are the estimates of the daily intakes of calcium in milligrams for 38 randomly selected women between the ages of 18 and 24 years who agreed to participate in a study of women's bone health.

808	882	1062	970	909	802	374	416	784	997
651	716	438	1420	1425	948	1050	976	572	403
626	774	1253	549	1325	446	465	1269	671	696
1156	684	1233	748	1203	1433	1255	1100		

a) Construct a 99% confidence interval for the true mean daily calcium intake for women ages 18-24.

State: We want to estimate μ, the true mean daily calcium intakes for all women between the ages of 18 to 24.

Plan: One-sample t confidence interval ($\sigma\ unknown$) with a 99% Confidence level

1) Assume that the sample is an SRS. The problem state that the subjects were randomly selected.

2)  The boxplot is relatively symmetric. There is a slight skew, but with no outliers and a sample size of 38, we can assume that the sampling distribution is approximately Normal.

3) Because the researchers samples without replacement, assume that there are at least 10(38) = 380 women between the ages of 18 to 24 in the population. Also assume that the calcium intake of each woman in the study was independent of the others.

Calculations:

Degrees of freedom: 37

CI = $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ = CI = $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ = $881.3 \pm\ (2.715)\frac{318.09}{\sqrt{38}}$ = (741.17, 1021.4)

Interpretation:

We are 99% confident that the true mean daily calcium intake for all women between the ages of 18 to 24 is between 741.17 and 1021.4 milligrams.

b) A nutritionist believes that the average calcium intake for women ages 18 to 24 is 739 milligrams per day. If we were to test this claim based on the confidence interval above, state the null and alternative hypothesis and the alpha level, then state your conclusions.

H_0: $\mu = 745$
H_A: $\mu \neq 745$
Alpha = 0.01

Because the null hypothesized value of 739 milligrams per day is not included within our interval, we can reject the null hypothesis at the 1% level. We do have evidence that the average calcium intake is not 739 milligrams (we can refute the nutritionists claim).

2) The composition of the earth's atmosphere may have changed over time. One attempt to discover the nature of the atmosphere long ago studies the gas trapped in bubbles inside ancient amber. Amber is tree resin that has hardened and been trapped in rocks. The gas in bubbles within amber should be a sample of the atmosphere at the time the amber was formed. Measurement on specimens of amber from the late Cretaceous era (75 to 95 million years ago) gives these percents of nitrogen:

63.4 65.0 64.4 63.3 54.8 64.5 60.8 49.1 51.0

Are these values significantly less than the present 78.1% of nitrogen in the atmosphere? Assume (this is not yet agreed on by experts) that these observations are an SRS from the Cretaceous atmosphere.

State: μ: the true mean percent of nitrogen in the earth's atmosphere during the Cretaceous era.

H_0: $\mu = 78.1$
H_A: $\mu < 78.1$

Plan: One-sample t test ($\sigma\ unknown$)

1) Assume that the sample is an SRS. (Stated in the problem)

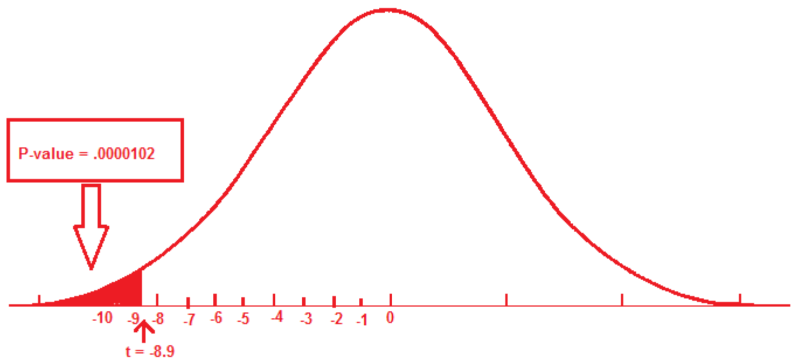
2)  The boxplot of the sample data is skewed. With a sample size of only 9, this skew may indicate a non-Normal the sampling distribution. The normality condition has not been met, so we will proceed with caution.

3) Assume that the measurements taken were independent of one another.

Calculations:

Degrees of freedom = 8

t = $\frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{59.59-78.1}{6.26/\sqrt{9}}$ = -8.878



p-value= 0.0000102

Interpretation:

There is a 0.00102% change of getting a sample mean as extreme as 59.59% if the population mean is 78.1%. Because this is VERY unlikely to occur, we can reject our null hypothesis. We have good evidence that the true mean percent of nitrogen in the earth's atmosphere during the Cretaceous era is less than 78.1%.

3.) White blood cell counts are normally distributed with mean 7500. If a patient has taken 50 laboratory blood tests that have a mean of 7312.5 and a standard deviation of 393.44, does this give evidence at the 1% level that his white blood cell count is significantly different than normal?

State: We want to estimate μ, the true mean white blood cell count for all patients.

H_0: $\mu = 7500$

H_A: $\mu \neq 7500$

Plan: One-sample t test (σ *unknown*)

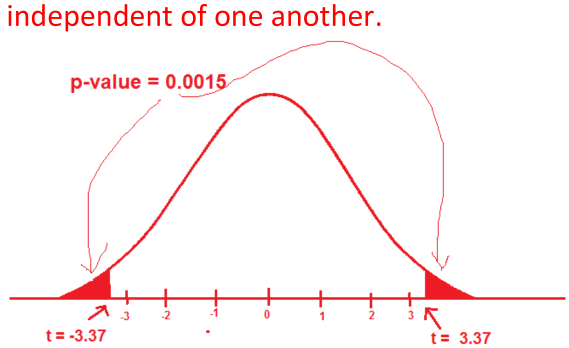
1) Assume that the sample is an SRS. The problem did not state anything regarding sampling methods.

2) We are told the population of white blood cell counts is normally distributed.

3) Assume that the individual measurements were independent of one another.

Calculations:

t = $\frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{7312.5-7500}{393.44/\sqrt{50}}$ = -3.37



df = 49

Conclusion:

Because our p-value 0.0015 is less than our significance level of 0.01, we can reject H_0. There is evidence that the true mean white blood cell count for all patients is not 7500.

4) For each of the following, decide if it describes 1 sample, 2 independent samples, or 2 dependent samples?

a) We are testing to see if the mean volume of a bag of regular m&m's is equal to the stated volume of 8 ounces. ONE SAMPLE

b) We are testing to see if the mean volume of a bag of regular m&m's is equal to the mean volume of a bag of peanut m&m's. TWO INDEPENDENT SAMPLES

c) We are testing to see if there is a preference of regular m&m's or generic chocolate candies by doing a blind taste test in which subjects eat both kinds of chocolate candies in a randomly selected order, and rank both on a scale from 1-5. 2 DEPENDENT SAMPLES (matched pairs)

5) An experiment was done by 15 students in a statistics class at the University of California at Davis to see if manual dexterity was better for the dominant hand compared to the nondominant hand (left or right). Each student measured the number of beans they could place in a cup in 15 seconds, once with the dominant hand and once with the nondominant hand. The order in which the two hands were measured was randomized for each student.

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Dominant hand	22	19	18	17	15	16	16	20	17	15	17	17	14	20	26
Nondominant hand	18	15	13	16	17	16	14	16	20	15	17	17	16	18	25
Difference	4	4	5	1	-2	0	2	4	-3	0	0	0	-2	2	1

a) Explain why the order of the two hands was randomized rather than, for instance, having each student tests the dominant hand first. If the order was not randomized, the order could be a confounding variable. For example, if every student did the test with their non-dominant hand first, the practice/experience could explain why they did better when they used their dominant hand.

b) Compute a 90% confidence interval for the mean difference in the number of beans that can be placed into a cup in 15 seconds by the dominant and nondominant hands. FULL PROCESS!

State:

μ: the true mean difference in number of beans that can be placed into a cup in 15 seconds (dominant hand – nondominant hand)

Plan: One sample t confidence interval (σ *unknown*)

1) Assume that the sample is an SRS. The problem did not state anything regarding sampling methods. The order of the two hands was randomized.

2) The boxplot shows a slight skew, but with a sample size of 15 we can say that the normality condition had been met.

3) Assume that the individuals' manual dexterity measurements were independent of one another. Because the researchers sampled without replacement, assume that there are at least 10(15) = 150 students in the population.

Calculations:

Degrees of freedom: 14

CI = $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ = CI = $1.067 \pm t^* \frac{2.434}{\sqrt{15}}$ = (–0.0402, 2.1735)

Interpretation:

We are 90% confident that the true mean difference in number of beans that can be placed into a cup in 15 seconds (dominant hand – nondominant hand) is between –0.0402 and 2.1735.

c) Use the interval to address the question of whether manual dexterity is better, on average, for the dominant hand.

H_0: $\mu_{diff} = 0$

H_A: $\mu_{diff} > 0$

Because 0 is included in the interval, we cannot reject our null hypothesis. We do not have evidence that the true mean difference in number of beans that can be placed into a cup in 15 seconds (dominant hand – nondominant hand) is greater than 0. (We don't have evidence that manual dexterity is better, on average, for the dominant hand.

6) In a study of memory recall, eight students from a large psychology class were selected at random and given 10 minutes to memorize a list of 20 nonsense words. Each was asked to list as many of the words as he or she could remember both 1 hour and 24 hours later. Is there evidence to suggest that the mean number of words recalled after 1 hour exceeds the mean recall after 24 hours **by more than 3 words**? Use .01 significance level.

Student	1	2	3	4	5	6	7	8
1 hr later	14	12	18	7	11	9	16	15
24 hours later	10	4	14	6	9	6	12	12
Difference	4	8	4	2	2	3	4	3

State: μ_{diff}: the true mean difference in number of word recalled after one hour vs. 24 hours (1 hr – 24 hrs)

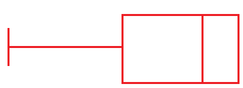
H_0: $\mu_{diff} = 3$

H_A: $\mu_{diff} > 3$

Plan: One-sample paired t test (σ *unknown*)

1) The problem stated that the sample was selected at random.

2) We are not told the shape of the population.



The boxplot is skewed and there is an outlier. Our sample size of 9 is not large enough to overcome these serious non-normal features. Proceed with caution.

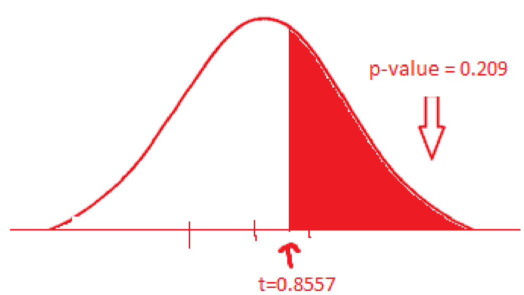
3) Because we sampled without replacement, assume that there are at least 10(8) = 80 students in the population. Also assume that the difference in the number of words remembered by each student was independent of other students.

Calculations:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3.625 - 3}{2.0659/\sqrt{8}} = 0.85569$$

df = 8

p-value = 0.209



Interpretation:

Because our p-value 0.209 is larger than our significance level of 0.01 we fail to reject H_0. There is not sufficient evidence that the true mean number of words recalled after 1 hour exceeds the mean recall after 24 hours by more than 3 words.

7)

Paired t for height – momheight

	N	Mean	StDev	SE Mean
Height	93	64.342	2.862	0.297
Momheight	93	63.057	2.945	0.305
Difference	93	1.285	3.136	0.325

95% CI for mean difference: (0.639, 1.931)
t-Test of mean difference = 0 (vs > 0); t-Value = 3.95; p-Value = 0.000

a) It has been hypothesized that college students are taller than they were a generation ago and therefore that college women should be significantly taller than their mothers. State the null and alternative hypotheses to test this claim. Be sure to define any parameters you use.
μ_{diff}: the true mean difference in heights of college students and their mothers (current students – mothers)

H_0: $\mu_{diff} = 0$
H_A: $\mu_{diff} > 0$

b) Using the information in the Minitab output, the test statistic is t = 3.95. Identify the numbers that were used to compute the t-statistic. What formula was used to calculate the t-statistic?

t = $\dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ $\quad\quad$ $\bar{x}_{diff} = 1.285, n = 93, s_{diff} = 3.136$

c) What are the degrees of freedom for the test statistic?
df = 92

d) Write the probability statement of the hypothesis test.
There is almost a 0% chance of getting a sample mean difference in heights as extreme at 1.285 due to sampling variability if the true mean difference in heights is 0. Because this is so unlikely, we can reject our null hypothesis at any reasonable significance level. We have strong evidence that the true mean height of college students is greater than the generation before.

e) Why is this a paired t test?
College students were paired with their mothers. Therefore, the two samples were not independent.