Estimating a Population Mean

- ✓ STATE and CHECK the Random, 10%, and Normal/Large Sample conditions for constructing a confidence interval for a population mean.
- ✓ EXPLAIN how the t distributions are different from the standard Normal distribution and why it is necessary to use a *t* distribution when calculating a confidence interval for a population mean.
- ✓ DETERMINE critical values for calculating a *C*% confidence interval for a population mean using a table or technology.
- ✓ CONSTRUCT and INTERPRET a confidence interval for a population mean.

Question --

On average, how long can a junior/senior in high-school hold his/her breath? 6
On average, how many pets do NP jrs/seniors have? 5
On average, how long can NP jrs/srs hold a plank?! 7
On average, how many minutes of exercise to NP jrs/srs get per day?

We know that a confidence interval has the following form:

Statistic ± (critical value) (standard deviation of the statistic)

$$\bar{x} \pm t^*\left(\frac{s}{\sqrt{n}}\right)$$

one sample t-interval for means

The *t* distributions

When we substitute the standard error of $\bar{x}$ ($\frac{s}{\sqrt{n}}$) for its standard deviation ($\frac{\sigma}{\sqrt{n}}$) we get the distribution of the resulting statistic, *t*.

We call it the ***t distribution***.

The t-statistic was introduced in 1908 by William Sealy Gosset, a chemist working for the Guinness brewery in Dublin, Ireland ("Student" was his pen name). Gosset devised the *t*-test as a way to cheaply monitor the quality of stout.

The *t* distributions

There is a **different** *t*-distribution for each sample size *n.*

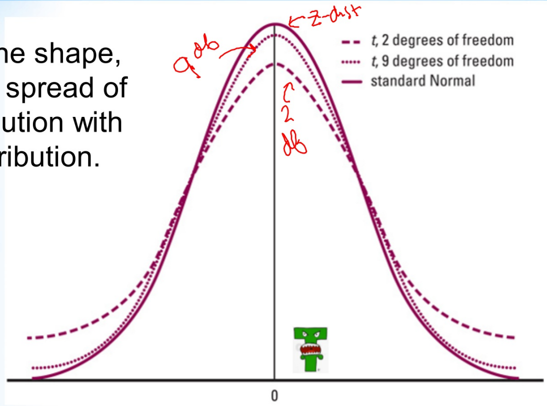
We specify a *t* distribution by giving its ***degrees of freedom***, which is equal to *n-1*

We will write the *t* distribution with *k* degrees of freedom as *t(k)* for short.

t(6)

Comparing *t* and *z* distributions

Compare the shape, center, and spread of the t-distribution with the z-distribution.



The One-Sample t Interval

Draw an SRS of size n from a population having unknown mean μ. A level C confidence interval for μ is

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

where t^* is the critical value for the $t(n-1)$ distribution. This interval is exactly correct when the population distribution is Normal and is approximately correct for large n in other cases.

Finding t with Table C

Suppose you want to construct a 95% confidence interval for the mean μ of a population based on a SRS of size n=10. What critical value t should you use?

df=9

$t^*=2.262$

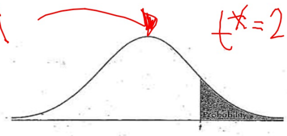


TABLE B: t-DISTRIBUTION CRITICAL VALUES

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	$\boxed{2.262}$	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Finding t with Table C

Suppose you want to construct a 99% confidence interval for the mean μ of a population based on a SRS of size n=39. What critical value t should you use?

df= 38

$t^*=2.750$

	Tail probability p								
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576
	50%	60%	70%	80%	90%	95%	96%	98%	99%
	Confidence level C								

Suppose you want to construct a 80% confidence interval for the mean μ of a population based on a SRS of size n=95. What critical value **t** should you use?

a) 1.290
b) .846
c) 1.292
c) .845

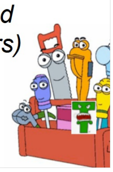
One sample t interval for μ

1)Random Sample/Random Assignment

2) 10% Rule

3) Large/Counts Normality (if you have the raw data you must draw a boxplot!!!)

- ***n* < 15** : *Use t procedures if data are close to Normal with* ***no outliers***
- ***n* ≥ 15** : *Use t procedures except in cases of outliers or strong skew*
- ***n* ≥ 30** : *Use t-procedures even for clearly skewed distributions (cannot have extreme outliers)*



One sample t interval for μ

Let's use our class data to construct a 95% confidence interval for the true mean

One sample t interval for mu

Step 1: STATE

Step 2: PLAN

Step 3: DO

Step 4: CONCLUDE